MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 99 No. 7

MEETING OF 1939 MAY 12

Professor H. C. Plummer, President, in the Chair.

The Society met in the Rooms of the Royal Society.

The election by the Council of the following Fellows was duly confirmed :—

Robert Atkinson, R.N.R., Dene House, Coble Dene, North Shields ;
Herbert Henry John Barton, 48 Cambridge Drive, Lee, London, S.E. 12 ; and
Joseph Jones, M.Sc., Headmaster, County School, Fishguard, Pembs.

The following candidates were proposed for election as Fellows of the Society, the name of the proposer from personal knowledge being appended :—

Miss Lydia Alberta Brown, 303 Bath Road, Hounslow West, Middlesex (proposed by A. Fowler) ;
H. S. W. Massey, B.A., M.Sc., Ph.D., Professor of Mathematics, University College, London, W.C. 2 (proposed by S. Chapman) ;
Allan Richard Dalhousie Ramsay, Marlborough College, Wilts. (proposed by R. v. d. R. Woolley) ; and
Rev. Wilfred Rees Wright, M.Sc., 21 St. Mark's Road, Salisbury, Wilts. (proposed by F. J. Sellers).

The President announced that the Council had elected the following as Associates of the Society :—

Hans Georg Kienle, Professor of Astronomy and Director of the University Observatory, Göttingen, Germany ; and
Georges Lemaitre, Professor of Mathematics in the University of Louvain, Belgium.

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Seventy-eight presents were announced as having been received since the last meeting.

The Secretary announced that owing to an increase in the cost of reproducing the Society's photographs for sale to Fellows, the price of prints and slides in future would be 1s. 6d. and 1s. 3d. respectively, instead of 1s. each as heretofore.

M. Lyot delivered the George Darwin Lecture,* taking as his subject "The Study of the Solar Corona and Prominences without Eclipses." The Lecture was illustrated by a film and by lantern slides.

At the request of the President, the Astronomer Royal expressed the thanks and appreciation of the audience to M. Lyot.

Before the Lecture, the President presented M. Lyot with the Gold Medal of the Society, which had been awarded to him for his observations and photography of the Solar Corona in the absence of a total eclipse. The President said :—

As you are all aware, the Gold Medal of our Society was awarded earlier in the year to M. Bernard Lyot. This evening it is our proud privilege to welcome him among us.

This is not the first time that we have met outside our own apartments, for the first meetings of our Society (in 1820) were held in the house of the Geological Society in Bedford Street. But it must be many years since an Ordinary Meeting has been held elsewhere than in our own premises. For the more ample accommodation we are enjoying on the present occasion we are indebted to the Royal Society, and it is my first duty to express on your behalf our gratitude and appreciation for this act of neighbourly hospitality.

Until twelve years ago it was the rule for your President to deliver a formal address annually when presenting the Gold Medal to the recipient or to his diplomatic representative. Since the institution of the George Darwin lecture by the generosity of Sir James Jeans, it has been more usual for the Medallist to deliver this lecture on the subject of his own work. On such an occasion, as happily on the present, it becomes superfluous for the President to explain in any detail the work for which the Council has awarded the Gold Medal. But he may be allowed to refer briefly to those antecedent events in the history of our science, which have no doubt weighed with the Council in making their award.

When the antiquity of Astronomy as a science in the true sense is remembered, and it is considered that the Sun is more important to mankind than all the other celestial bodies put together, it is a most remarkable thing that solar research is a completely modern development. In favourable circumstances sunspots can be seen with the naked eye, and ancient Chinese records refer to them as "flying bird seen in the Sun". Such observations are too irregular and sporadic for any scientific deductions. But from the beginning of the seventeenth century it has been possible to make systematic

* It is hoped to publish the Lecture in the June number of *Monthly Notices*.

observations of the Sun's disc, and much might have been found out. Yet it was only in 1842 that Schwabe was able to announce the periodicity of sunspots, ten years later that Sabine noted the connection with terrestrial magnetism, and in 1855 that Schwabe received the Gold Medal. That was the first time that this medal was awarded for Solar Research.

The reason why advance on a broader front was still impossible is clear enough. That was waiting for the development of photography and the aid of spectrum analysis. But there is another circumstance which ought not to be ignored. The first advance, when it came, was largely associated with the observation of total eclipses. These opportunities had been almost completely neglected by astronomers till half-way through the nineteenth century. Francis Baily, one of the founders of our Society and a distinguished predecessor in this chair, is a notable exception.

There is a point here which deserves careful attention. When we consider the great cost of eclipse expeditions, not only in money but in the time and labour spent in the work of preparation, and when we know that all efforts and all plans may prove abortive, and that at the best the harvest is limited to a few precious minutes, some of us have been tempted to wonder whether, to put it bluntly, the game was worth the candle. This sceptical view should not be accepted too lightly. It is not merely that the total addition to our knowledge gathered by eclipse expeditions has been by no means small. It has been, on the contrary, considerable. But the effect of eclipse work is not to be estimated in that way. It has resulted in setting the problems which have guided solar research in its normal development. Without the stimulus afforded by these problems it is likely that the progress achieved would not have followed the same paths, that it would have been far slower and that it would have been less rich in results.

Brief reference may be made to three of these problems, familiar as they are to all of you. The first is that presented by the prominences, at first an eclipse phenomenon exclusively, of which the nature was only discovered in the late eighteen-sixties. When, in 1868, Janssen and Lockyer simultaneously realized the possibility of observing a prominence at any time, the Paris Academy of Sciences showed a fitting sense of the occasion by the famous award of twin medals. For this marked a true epoch in solar research. The invention of the spectroheliograph followed after an interval in logical sequence, with all the developments due to Hale and to M. Deslandres. The latest advances have been made in the United States at the McMath-Hulbert Observatory and in France by our Medallist. Of the American researches, Dr. H. D. Curtis has only recently given us an account. This evening we shall have the privilege of hearing from M. Lyot himself the story of his work in this field.

A second problem was set by the chromospheric spectrum recorded at second or third contact by the objective prism in the form of the flash spectrum. Later a more refined technique was introduced by Campbell for the same object. To pass in this case from eclipse to ordinary conditions the requirements are fairly obvious. It is only necessary to have a large image of the Sun, coupled with perfect definition, so that the slit of the

spectrograph can be set with certainty beyond the edge of the photosphere. These conditions were first obtained with the tower telescope on Mount Wilson, and the problem is only mentioned as an illustration of the continued effort to extend the range of observation beyond the precarious and fleeting opportunities presented by a total solar eclipse into the region where the research can be pursued systematically at all times.

But after it had become possible to observe and record the solar prominences from day to day, and to investigate the conditions of the lower atmosphere of the Sun at all times, one problem remained to furnish M. Lyot with his most impressive achievement. Down to 1850 it was still uncertain whether the corona was a truly solar phenomenon, or whether it was an appearance within our own atmosphere. After that time it was successfully observed by direct photography and with the help of the spectroscope and the polariscope. But this work was strictly confined to the brief intervals when the Sun was totally eclipsed. Was it likely that this limitation would ever be removed?

It is not that the corona is a particularly faint object. The full Moon only furnishes about twice as much light. But the uneclipsed Sun furnishes about a million times as much. The case has some analogy with that of Jupiter's principal satellites, which would be easily visible to the naked eye were it not for the presence of Jupiter. Except during a total eclipse the corona must be seen through an atmosphere illuminated by scattered sunlight, and, furthermore, the sunlight which falls directly on the optical instrument employed can hardly be prevented from entering with such diffused intensity as to obliterate the effect of a fainter image.

The conditions of the problem were such as to appal all but the boldest spirits. Nevertheless it did not go unattacked. Huggins made the attempt and failed. M. Deslandres was no more successful. Hale brought all his resources, all his ingenuity and all his persistence to bear on the problem, and the result was the same. It seemed as if the enabling power to see the corona was restricted to the eclipsing Moon.

Here in this field where the most skilful astronomers have failed in their endeavours, M. Lyot has achieved his most impressive triumph. It is unnecessary to say more because he will himself show us the results he has obtained in the course of his work and will explain the technical means by which those results have been won. It will be recognized that our Medallist of the year is a worthy successor to men like Janssen and Deslandres, who have made the French school of solar physicists a part of the intellectual glory of their country.

M. Lyot, on behalf of the Royal Astronomical Society I have the honour to present to you this Gold Medal which represents the highest distinction it is in our power to confer. You will receive it as a token of our admiration for the work you have accomplished, and our sincere wishes that you may long be given the health and strength to continue your researches with equal success. Will you allow me to add my personal congratulations.

And now, M. Lyot, I invite you to deliver the George Darwin lecture for the year 1939.

THE ROTATION OF THE EARTH, AND THE SECULAR ACCELERATIONS OF THE SUN, MOON AND PLANETS

H. Spencer Jones, F.R.S., Astronomer Royal

1. It has been fully established by the researches of various investigators that there are fluctuations in the longitudes of the Sun, Mercury and Venus, which run closely parallel to the fluctuations in the longitude of the Moon. The fluctuations in the longitudes of these several bodies have therefore been attributed to a common cause, a variation of the adopted unit of time provided by the rotation of the Earth.

Variations in the rotation of the Earth have been attributed to two causes :

(*a*) Tidal friction, particularly in enclosed or narrow seas.

(*b*) Changes in the moment of inertia of the Earth, as the result of some redistribution of matter within the Earth.

Tidal friction can cause only a retardation of the rotation, never an acceleration. The portions of the secular accelerations of the longitudes of the Sun and Moon, not explained by gravitational theory, are attributed to the slowing down of the Earth by tidal friction.

Changes in the moment of inertia of the Earth may result in acceleration or retardation of the rotation, according as the moment of inertia is decreased or increased. The irregular fluctuations in longitude are attributed to this cause.

The effects produced by the two causes differ in one respect. If there is a change in the moment of inertia of the Earth, the effects on the longitudes of the Sun, Moon and planets, when expressed in time are identical or, in other words, the effects on the longitudes, when expressed in arc, are proportional to the mean motions. Associated with the retardation of the Earth by tidal friction there is a change of the angular momentum of the Earth, which must be compensated by a corresponding change in the angular momentum of the Moon's orbital rotation (the angular momentum associated with the axial rotation of the Moon can be neglected). Tidal friction therefore produces a change in the true mean motion of the Moon. The effects on the longitudes of the planets will be identical in time, or, otherwise expressed, the effects in angle will be proportional to the mean motions ; but the effect on the longitude of the Moon, bound up as it is with changes in the shape of the Moon's orbit, cannot be predicted by theory.

These expectations, though not so fully substantiated by observation as is desirable, have been generally confirmed with one exception, referred to in section 3.

2. For the purpose of discussion of the observational material, it is

necessary to formulate an expression for the fluctuation of the longitude of the Moon, or, in other words, for the difference between the true longitude and the longitude according to pure gravitational theory. In Brown's Tables of the Moon, the great empirical term of Newcomb was included in order to secure approximate agreement with observations and the adopted value of the secular acceleration of the Moon was the theoretical value, produced by the attractions of the planets. Brown considered the total secular acceleration was not sufficiently well determined by observation to be used in his tables.

The investigations of Fotheringham, summarised and rediscussed by de Sitter, lead to a value of $5''\cdot22$ as best representing the average value of the residual secular acceleration of the Moon, not accounted for by gravitational theory, over a period of about two thousand years. The question whether this value can be accepted as representing modern observations will be discussed subsequently.

We write, therefore,

$$\begin{aligned} B &= \text{Observed Longitude} - C, \\ C &= \text{Brown's Tables} - 10''\cdot71 \sin(140^\circ\cdot0T + 240^\circ\cdot7) \\ &\quad + 5''\cdot22T^2 + 12''\cdot96T + 4''\cdot65, \end{aligned}$$

and define the quantity B as the *fluctuation* in the Moon's longitude. We have removed the great empirical term, applied a correction to reduce from the theoretical secular acceleration adopted by Brown to the observed value and made consequential corrections to the mean motion and longitude at epoch (which are purely observational quantities) in order to secure close agreement with modern observations.

It will be noted that we have separated the secular acceleration term from the fluctuation. We have, in effect, represented the difference between observation and pure gravitational theory by an expression of the form

$$\alpha + \beta T + 5''\cdot22T^2 + B.$$

The question whether the fluctuations and the secular acceleration can be uniquely separated will be discussed subsequently.

3. We will assume that the correction required by the longitude of the Sun according to Newcomb's tables may be expressed in the form

$$\Delta L = a + bT + cT^2 + Q \cdot \frac{n_0}{n} B$$

and by the planets Mercury and Venus to be

$$\Delta l_i = a_i + b_i T + c_i T^2 + Q \cdot \frac{n_i}{n} B,$$

where $i = 1$ for Mercury and $= 2$ for Venus. n denotes the mean motion of the Moon, n_0, n_1, n_2 denote the mean motions of the Sun, Mercury and Venus respectively. T is expressed throughout in centuries from 1900·0.

The quantity Q was introduced by de Sitter * to take account of the possibility that the terms involving B in the longitudes of Moon, Sun and planets were not strictly proportional to the mean motions and it was not assumed that Q had the same value for these bodies. The discussions by de Sitter and the present author † agree in establishing that, within the limits of observational error Q has the same value for the Sun, Mercury and Venus. The value given by de Sitter's discussion was 1·25; the value found by the present author was 1·19. The observational data used for the Sun, Venus and Mercury were essentially the same in the two discussions, but de Sitter used the results derived by Newcomb from eclipses and occultations up to 1835 and the Greenwich meridian observations after that date, whereas my discussion was based entirely on my revision of Newcomb's discussion of occultation observations, supplemented by more recent data.

There are great difficulties in interpreting a value of Q that is greater than unity and de Sitter's suggested explanation, involving sudden and very great changes in tidal friction, is artificial and not convincing. The main purpose of the present discussion, which has used additional observational material, and has proceeded along somewhat different lines was the reinvestigation of the value of Q. The result of this discussion is to show that Q has the value unity within the limits of probable error.

The previous discussions have also shown that within the limits of observational error we may write

$$c_1 = n_1 c/n_0 \quad \text{and} \quad c_2 = n_2 c/n_0.$$

These relationships, implying that the secular accelerations are proportional to the mean motions, are only established for the Sun and Mercury; for Venus the results that are available for use and sufficiently free from accidental error do not cover a time long enough to determine the secular acceleration of Venus with much accuracy. The Venus observations do not, however, contradict the relationship.

4. The previous discussions by de Sitter and myself were based on the following data: for the Sun, the data given by me ‡ for the period 1835 to 1923, which de Sitter supplemented by data for the period 1750 to 1834, taken from the paper by E. W. Brown §; these early observations were so erratic that they are not of much value and may seriously impair the results. For Venus, the data given by me ‡ for the period 1836 to 1923. For Mercury, the observations of the transits across the Sun, discussed by Newcomb || and rediscussed by Innes ¶ were used; these extend from 1677 to 1924.

The longitude errors of the Sun that were used were based entirely on observations of right ascension. These observations are peculiarly liable to systematic errors, depending on the observer and the method of observation. The observations at Greenwich, for instance, in which several

* *B.A.N.*, 4, 21, 1927. † *Cape Annals*, 13, part 3, 1932 (see § vii).
‡ *M.N.*, 87, 4, 1926.
§ *Transactions of Yale University Observatory*, 3, part 6, 1926.
|| *Astr. Papers American Ephemeris*, 1, part vi, 363, 1882.
¶ *Union Observatory Circular*, No. 65, 1925.

observers participate, show some considerable personal differences, which do not necessarily remain constant for long periods of time ; changes are necessarily introduced by changes in the rota of observers. The change from the eye-and-ear to the tapping method of recording, and from the tapping method to the use of the travelling wire micrometer involve discontinuities, which are not easy to assign accurately. It is accordingly impossible to be certain that the corrections to the Sun's longitude from Newcomb's tables, as inferred from observations of right ascension, are on a uniform basis.

The data for the Sun's longitude are specially important, on the other hand, for the present purpose, because they enter into the reduction of the observations of Venus and Mercury. In the reduction of the Venus observations, corrections to the Sun's tabular longitude, read off from smooth curves drawn to represent as closely as possible the observed values, were used ; any systematic errors in the observations of the Sun must consequently affect the observations of Venus. The same procedure was not possible in dealing with the observations of the transits of Mercury, because these cover a much longer period than the available Sun observations. The expression derived for the representation of the Sun observations was therefore extrapolated for the much longer period of the transit of Mercury observations. These are therefore also affected by any systematic errors in the Sun observations, with the possibility that by extrapolating backwards in time their influence may have been materially increased.

Minor defects in treatment were the use of smoothed values for the longitudes of the Sun and Venus ; the smoothing may introduce systematic effects. Equal weight was given to each value read off from the smoothed curve drawn to represent the observations ; the earlier observations were therefore considerably overweighted.

5. The longitude of the Sun can be derived from observations of either declination or right ascension ; declination observations, in the case of the Sun, are far more likely to be free from systematic errors than are the observations of right ascension. Though the correction to the Sun's tabular longitude can be inferred with much greater weight from right ascension observations than from declination observations, the declination observations are likely to give a much truer representation over a long period of time of the changes in the correction. The reduction in weight, as inferred from the internal accordance of the observations during a limited period, is more than counterbalanced by the greater freedom from errors of a systematic nature. In the present discussion, observations of declination have been utilised and have been accepted as likely to provide the best representation of the Sun's longitude. The smaller systematic error of the declination observations is supported by the fact that the longitudes deduced from the early observations of declination do not show the large and irregular variations that characterise the longitudes deduced from the observations, during the same period, of the right ascensions. It has been possible to incorporate the observations of declination back to the year 1750 with substantial advantage.

It was also decided not to derive longitudes of Venus and Mercury by using either smoothed values of the Sun's longitude or a representation of the observed longitudes by a formula, but to make direct use of the quantities actually furnished by the observations.

The observations of Venus give directly the values of $\Delta l_2 - \Delta L$, *i.e.* the difference of the longitude corrections of Venus and the Sun. The observations of the transits of Mercury give the quantities denoted by V and W by Newcomb *, viz.:

November transits : $V = 1{\cdot}487\Delta l_1 - 1{\cdot}01\Delta L +$ corrections to other elements.

May transits : $W = 0{\cdot}716\Delta l_1 - 0{\cdot}97\Delta L +$ corrections to other elements.

The observations were reduced in the following way. It is assumed that the corrections to the longitudes of the Sun, Mercury and Venus may be represented by the following expressions :—

$$\Delta L = a + bT + cT^2 + Q(\cdot 0747)B, \tag{1}$$

$$\Delta l_1 = a' + b'T + 4{\cdot}15cT^2 + Q(\cdot 310)B, \tag{2}$$

$$\Delta l_2 = a'' + b''T + 1{\cdot}63cT^2 + Q(\cdot 122)B. \tag{3}$$

The ratios of the mean motions of Mercury and Venus to the mean motion of the Sun are respectively 4·15 and 1·63.

By this representation it is assumed that the secular accelerations of the Sun and planets, arising from the retardation of the Earth by tidal friction, are proportional to their respective mean motions. No assumption is made about the ratios of these secular accelerations to the secular acceleration of the Moon. It is also assumed that the fluctuations in the longitude of the Moon are reflected in the longitudes of the Sun and planets proportionally to their mean motions. If, as is to be expected on theoretical grounds, these fluctuations are due to change of moment of inertia of the Earth, the ratio between the effects on the longitudes of Sun and Moon should be proportional to the mean motions ; or, since the ratio of the mean motions is ·0747, we should expect that Q would have the value unity.

Observations give the following expressions :—

Mercury : November Transits.—The corrections to the other elements of the orbit of Mercury may be assumed to be represented by an expression of the form $x + yT$. Hence we may write

$$V = a_1 + b_1T + 5{\cdot}16cT^2 + Q(\cdot 385)B. \tag{4}$$

Mercury : May Transits.—In a similar way we may write

$$W = a_2 + b_2T + 1{\cdot}97cT^2 + Q(\cdot 147)B. \tag{5}$$

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* *Astr. Papers American Ephemeris*, 1, part vi, 447, 1882.

Venus : Observations give the quantity $\Delta l_2 - \Delta L$ which we may write in the form

$$\Delta l_2 - \Delta L = a_3 + b_3 T + 0{\cdot}63 c T^2 + Q(\cdot 0471)B. \qquad (6)$$

It will be seen that the Venus observations are of much less value than the observations of Mercury or the Sun for the determination of c and Q.

The observations of Sun, Mercury and Venus were combined for the determination of c and Q.

6. *The Observational Material.* (a) *The Fluctuations in the Moon's Longitude.*—The values of the fluctuations, derived in *Cape Annals*, **13**, part 3 (Table VI, p. 31) have been used. These have been supplemented for recent years by the results derived from the errors of the Moon's longitude according to Brown's tables, as deduced from extensive observations of occultations. The reductions of these occultations have been published by Brown and Brouwer in various numbers of the *Astronomical Journal.* For convenience the fluctuations for the entire period covered by the observations that have been used are summarised in Table I.

TABLE I

Fluctuations in the Moon's Mean Longitude

Date	B	Date	B	Date	B
	″		″		″
1681·0	−12·72	1837·4	+ 4·91	1900·5	−15·87
1710·0	− 3·92	43·1	+ 4·31	03·5	−14·50
27·0	+ 2·15	48·8	+ 3·97	06·5	−13·43
37·0	+ 5·97	52·5	+ 3·37	09·5	−12·78
47·0	+ 8·49	57·5	+ 2·40	12·5	−11·62
55·0	+10·34	62·5	+ 0·91	15·5	−10·35
71·0	+13·54	67·5	− 1·57	18·5	−10·20
85·0	+14·84	72·5	− 6·38	21·5	−10·18
92·0	+14·53	77·5	− 9·38	24·5	−11·82
1801·5	+13·09	82·5	−11·31	26·5	−12·11
09·5	+11·80	87·5	−13·05	28·5	−12·90
13·0	+11·28	91·5	−14·34	30·5	−13·83
21·8	+10·02	94·5	−15·23	32·5	−14·81
31·5	+ 6·85	97·5	−15·99	34·5	−15·98
				36·5	−16·48

(b) *The Sun : Declinations.*—The material used for the Sun for the observations prior to 1900 was taken from the data given by Newcomb *, supplemented by the observations made by Hornsby at the Radcliffe Observatory from 1774 to 1798 † and the Greenwich Observations for 1893 to 1900. Corrections to reduce from Leverrier's tables to Newcomb's tables were applied, the data for this purpose being extracted from papers by Ross ‡ and myself.§

* *Astronomical Constants*, 30–32, 1895.

† Knox-Shaw, Jackson and Robinson, *Hornsby's Meridian Observations, 1774–1798*, 100–104, 1932.

‡ *Astronomical Journal*, **29**, 153, 1916.

§ *M.N.*, **86**, 428, 1926 ; *Cape Annals*, **13**, part 3, 33–34, 1932.

After 1900, the mean annual results given by the Greenwich observations up to the year 1936 were combined with the data for other observatories summarised by Morgan and Scott.*

The Greenwich series of observations made with the Airy transit circle was adopted as a basis and systematic corrections were applied to the older series of observations by comparisons of overlaps to reduce them to the same basis. The observations were weighted, an attempt being made to assign a series of weights such that unit weight corresponded to a probable error of 0″·50. The adopted mean values and weights are given in Table II, which includes also information about the observations included in each mean value.

TABLE II

Sun's Longitude (Observed minus Newcomb) from Declination

Mean Date	Correction to Tables	Weight	Observations	$B/10$	Residual (O − C)	Fluctuation
	″			″	″	″
1761·3	+ ·31	5	Gr	+1·33	+·09	+14·5
77·9	− ·01	9	Gr, H	+1·44	−·30	+10·5
89·8	+ ·32	6	Gr, H	+1·45	+·02	+14·7
99·0	+ ·68	6	Gr, H	+1·34	+·43	+19·2
1811·4	+ ·05	11	Gr, Pa	+1·15	−·13	+ 9·8
20·5	− ·49	9	Gr, Pa, Kb	+1·03	−·68	+ 1·2
25·8	+ ·22	15	Gr, Pa, Kb, Dp	+ ·87	+·09	+ 9·9
31·7	+ ·31	13	Gr, Kb, Dp	+ ·67	+·25	+10·0
37·8	− ·03	15	Gr, Pa, Dp, Ca	+ ·49	−·04	+ 4·4
42·9	+ ·15	17	Gr, Kb, Ca, Pu, Ox	+ ·43	+·13	+ 6·0
48·9	+ ·22	29	Gr, Pa, Ca, Pu, Ox, Wa	+ ·39	+·12	+ 5·5
56·7	+ ·12	15	Gr, Pa, Ca	+ ·24	−·01	+ 2·3
63·3	+ ·22	24	Gr, Pa, Pu, Ox, Wa	− ·06	+·20	+ 2·1
68·9	+ ·12	29	Gr, Pa, Pu, Ox, Wa, Lei	− ·26	+·11	− 1·1
74·6	− ·40	26	Gr, Pa, Ox, Wa, Lei	− ·79	−·14	− 9·8
81·1	− ·42	20	Gr, Pa, Ox, Wa	−1·07	−·11	−12·2
86·2	− ·29	23	Gr, Pa, Ox, Cp, Sb	−1·25	+·04	−11·9
89·5	− ·24	17	Gr, Wa, Cp	−1·37	+·07	−12·7
97·0	− ·29	15	Gr	−1·59	−·01	−16·1
1902·5	− ·16	30	Gr, Wa	−1·49	−·14	−16·7
08·2	+ ·23	90	Gr, Pu, Wa, Cp	−1·31	−·04	−13·7
13·1	+ ·60	75	Gr, Pu, Wa, Cp	−1·14	+·04	−10·9
17·0	+ ·52	37	Gr, Al	−1·03	−·25	−13·7
22·1	+ ·90	45	Gr, Wa, Be, Br	−1·05	−·03	−10·9
27·0	+1·08	37	Gr, Wa	−1·23	+·11	−10·9
30·4	+1·08	37	Gr, Wa	−1·38	+·10	−12·5
34·2	+1·05	30	Gr, Wa	−1·58	+·07	−14·9

In the above table the following abbreviations are used : Gr = Greenwich ; H = Hornsby (Radcliffe) ; Pa = Paris ; Kb = Königsberg ; Dp = Dorpat ; Ca = Cambridge ; Pu = Pulkova ; Ox = Oxford (Radcliffe) ; Wa = Wash-

* *Astronomical Journal*, **47**, 193, 1939.

ington ; Lei = Leiden ; Cp = Cape of Good Hope ; Sb = Strasbourg ; Al = Algiers ; Be = Berlin-Babelsberg ; Br = Breslau.

The fifth column of the table gives the values of $B/10$, where B is the quantity adopted to represent the fluctuation in the Moon's longitude. The residuals from the finally adopted solution are given in the sixth column. The last column of the table will be explained later.

(c) *The Sun : Right Ascensions.*—The data used in my previous discussion * were utilised, but mean values for each few years were taken. The Greenwich results have been extended to the year 1936 and the data for other observatories, from 1900 onwards, given by Morgan and Scott †, have been incorporated. The observations have been reduced to the basis of the Greenwich observations with the Airy transit circle, using the hand-tapper. A discontinuity of $0^s.030$ in the Greenwich results with the introduction of the travelling wire has been adopted.

The mean values together with the adopted weights, assigned on the basis of unit weight corresponding to a probable error of $0''.50$, are given in Table III together with the observatories whose observations have been incorporated in each mean value. The table is arranged similarly to Table II.

TABLE III

Sun's Longitude (Observed minus Newcomb) from Right Ascensions

Mean Date	Correction to Tables	Weight	Observations	$B/10$	Residual (O − C)	Fluctuation
	″			″	″	″
1839·0	+ ·34	7	Gr, Kb, Dp, Ca	+ ·46	− ·10	+ 3·2
47·4	+ ·54	7	Gr, Ca, Pu, Wa	+ ·41	+ ·02	+ 4·4
56·5	+ ·51	5	Gr, Ca	+ ·26	− ·02	+ 2·3
63·2	+ ·33	7	Gr, Wa	+ ·05	− ·17	− 1·7
68·3	+ ·29	14	Gr, Pu, Wa	− ·19	− ·12	− 3·5
73·5	+ ·37	5	Gr	− ·72	+ ·25	− 3·9
78·3	+ ·39	9	Gr, Wa	− ·97	+ ·35	− 5·0
83·0	+ ·20	5	Gr	− 1·15	+ ·20	− 8·8
87·2	− ·35	16	Gr, Cp, Wa, Sb	− 1·29	− ·35	− 17·6
92·5	− ·18	7	Gr, Pa	− 1·46	− ·18	− 17·0
96·5	+ ·23	13	Gr, Wa	− 1·58	+ ·22	− 12·9
99·5	+ ·13	13	Gr, Pa, Wa	− 1·61	+ ·07	− 15·1
1902·5	+ ·12	16	Gr, Pa, Wa, Pu	− 1·49	− ·12	− 16·5
05·7	+ ·24	16	Gr, Pa, Wa, Pu	− 1·36	− ·17	− 15·9
09·7	+ ·66	27	Gr, Wa, Cp, Pu	− 1·27	+ ·06	− 11·9
14·9	+ 1·04	45	Gr, Wa, Cp, Pu, Al	− 1·06	+ ·12	− 9·0
19·7	+ 1·14	32	Gr, Wa	− 1·02	+ ·04	− 9·6
24·0	+ 1·25	27	Gr, Wa	− 1·16	+ ·12	− 10·1
28·0	+ 1·24	18	Gr	− 1·27	+ ·06	− 11·9
32·2	+ 1·01	28	Gr, Wa	− 1·45	− ·19	− 17·0
35·3	+ 1·03	23	Gr, Wa	− 1·62	− ·14	− 18·1

* *Cape Annals*, **13**, part 3, 1932.

† *Astronomical Journal*, **47**, 193 (Table I), 1939.

(d) *Transits of Mercury.*—The data given by the rediscussion of the observations of the transits of Mercury by Innes, as summarised by de Sitter *, have been adopted without alteration. The only addition is the inclusion of data for the 1927 transit, compiled from the mean of all the observations that have been published. The transits of 1782 November and 1740 May, which are badly discordant, have been discarded.

The data are summarised in Table IV, which gives the observed values of V and W from the November and May transits respectively, the relative weights on the same basis as for the Sun observations, the quantity $B/10$, which is proportional to the fluctuations in the Moon's longitude, and the residuals from the final solution. The last column will be explained later.

TABLE IV

Transits of Mercury

(*a*) November Transits

Date	V	Weight	$B/10$	Residual (O − C)	Fluctuation
	″		″	″	″
1677·9	−2·57	0·1	−1·36	+1·11	−10·7
90·9	−2·32	0·1	−1·10	+1·83	− 6·2
97·9	−7·09	0·3	−0·91	−2·96	−16·8
1723·9	−1·10	1·0	+0·09	+1·24	+ 4·1
36·9	−0·42	1·7	+0·60	+0·67	+ 7·7
43·9	−0·86	1·9	+0·77	−0·14	+ 7·3
56·9	−0·11	0·2	+1·08	−0·21	+10·3
69·9	−0·50	0·9	+1·29	−1·20	+ 9·8
89·9	+1·38	1·7	+1·47	−0·16	+14·3
1802·9	+0·34	0·6	+1·29	−0·88	+10·6
22·9	+0·76	1·1	+0·98	−0·23	+ 9·2
48·9	+1·35	2·7	+0·46	+0·29	+ 5·4
61·9	+1·82	2·1	+0·13	+0·64	+ 3·0
68·9	−0·08	3·5	−0·25	−0·61	− 4·1
81·9	−1·23	4·0	−1·07	−0·22	−11·3
94·9	−0·47	4·3	−1·53	+0·41	−14·2
1907·9	+2·10	4·1	−1·31	−0·01	−13·1
14·9	+4·14	4·5	−1·06	−0·11	−10·9
27·9	+5·92	5·4	−1·27	+0·02	−12·7

(*b*) May Transits

Date	W	Weight	$B/10$	Residual (O − C)	Fluctuation
	″		″	″	″
1753·4	−0·78	2·0	+1·00	−0·18	+ 8·8
86·4	−0·18	7·5	+1·48	−0·45	+11·8
99·4	+0·32	7·0	+1·34	+0·05	+13·7
1832·4	+0·42	11·5	+0·65	+0·24	+ 8·2
45·4	+0·56	13·0	+0·42	+0·22	+ 5·7
78·4	−0·06	12·5	−0·97	+0·01	− 9·7
91·4	−0·05	12·5	−1·43	−0·10	−15·0
1924·4	+2·59	7·0	−1·18	−0·21	−13·2

* *B.A.N.*, 4, 31, 1927.

(e) *Venus.*—The data have been taken from my previous paper *, supplemented by more recent Greenwich observations. For the purpose of deriving longitude errors of Venus the final adopted smoothed errors of the Sun's longitude were used. For the present discussion the data are given in the form actually derived in the first instance, *i.e.* the values of $\Delta l_2 - \Delta L$ are tabulated. The material is summarised in Table V.

TABLE V

Longitude Difference (Venus minus Sun) : Observed minus Newcomb's Tables

Date	Long. Diff.	Weight	$B/10$	Residual	Fluctuation
	″		″	″	″
1839·0	− ·67	2	+ ·45	− ·49	− 5·9
45·4	+ ·13	3	+ ·42	+ ·21	+ 8·7
51·8	+ ·31	5	+ ·35	+ ·27	+ 9·1
58·2	+ ·41	5	+ ·23	+ ·31	+ 8·9
64·5	+ ·30	5	− ·02	+ ·20	+ 4·0
70·9	− ·14	5	− ·43	− ·21	− 8·7
77·3	+ ·25	5	− ·93	+ ·29	− 3·2
83·7	− ·45	6	− 1·17	− ·47	− 21·6
90·1	− ·34	8	− 1·39	− ·41	− 22·5
96·5	− ·17	6	− 1·58	− ·30	− 22·0
1902·9	+ ·52	7	− 1·47	+ ·15	− 11·4
09·3	+ ·64	9	− 1·28	+ ·01	− 12·5
13·5	+ ·72	10	− 1·12	− ·12	− 13·8
16·5	+ 1·13	9	− 1·03	+ ·16	− 7·0
20·0	+ 1·41	8	− 1·02	+ ·33	− 3·2
23·6	+ 1·52	8	− 1·13	+ ·37	− 3·4
26·6	+ 1·29	9	− 1·22	+ ·10	− 10·0
29·6	+ 1·39	9	− 1·34	+ ·16	− 10·0
33·1	+ 1·06	8	− 1·52	− ·18	− 19·1
35·5	+ ·80	6	− 1·62	− ·49	− 26·5

7. *Derivation of the Constants.*—The expressions (1), (4), (5) and (6) in section 5 were adopted to represent the observed data summarised in Tables II to V. Normal equations were formed, the whole of the material being combined for the quantities c and Q. Two solutions were made : in the first the data for the longitude errors of the Sun from right ascensions were excluded ; in the second they were included. The results of these solutions were as follows :—

I. *Sun's R.A.'s Excluded*

$a + 1''·02$	$a_1 + 5''·90$	$a_2 + 2''·78$	$a_3 + 1''·00$
$b + 3''·02$	$b_1 + 16''·26$	$b_2 + 6''·94$	$b_3 + 2''·72$
	$c = + 1''·25$	$Q = 1·025.$	

* *M.N.*, 87, 4 (Table VII), 1926.

II. *Sun's R.A.'s Included*

a, b denote the correction to the longitude at epoch and to the mean motion of the Sun derived from declination observations ; a', b' the corresponding quantities derived from right ascension observations.

$a+1''\cdot04$	$a'+1''\cdot32$	$a_1+\ 6''\cdot03$	$a_2+2''\cdot85$	$a_3+1''\cdot02$
$b+3''\cdot09$	$b'+2''\cdot67$	$b_1+16''\cdot55$	$b_2+7''\cdot10$	$b_3+2''\cdot76$

$$c=+1''\cdot26 \qquad Q=+1\cdot062.$$

The probable error of the value of Q is about $\pm\ \cdot033$. It will be seen, therefore, that when the right ascension observations of the Sun are excluded the value of Q is unity, within an amount that is less than the probable error. When the right ascension observations of the Sun are included the value of Q is increased ; it is considered that the increase has been caused by systematic errors in the right ascension observations of the Sun. It is accordingly concluded that the observations provide no justification for the assumption that Q does not have the value unity, and, moreover, as has already been mentioned, there are great difficulties in interpreting any other value.

It is accordingly assumed that we may write $Q=1$. The derived value of c is then practically unaffected by the inclusion of the right ascension observations ; these contribute very little weight to the determination of c because they extend over a relatively short period.

With this assumption we obtain—

III. *Adopted Solution*

$a+1''\cdot00$	$a'+1''\cdot28$	$a_1+\ 5''\cdot81$	$a_2+2''\cdot74$	$a_3+0''\cdot98$
$b+2''\cdot97$	$b'+2''\cdot69$	$b_1+16''\cdot01$	$b_2+6''\cdot82$	$b_3+2''\cdot70$

$$c=1''\cdot23 \qquad Q=+1\cdot00.$$

With these values the following probable errors corresponding to unit weight are obtained : for the declination observations of the Sun, $\pm\ 0''\cdot49$; for the right ascension observations of the Sun, $\pm\ 0''\cdot42$; for the November transits of Mercury, $\pm\ 0''\cdot50$; for the May transits of Mercury, $\pm\ 0''\cdot43$; for the observations of Venus, $\pm\ 0''\cdot48$. As an attempt was made to assign weights on the basis that unit weight corresponded to a probable error of $\pm\ 0''\cdot50$, these values may be regarded as satisfactory.

The fluctuations given in the last column of Tables II to V were derived from this adopted solution. In the expressions (1), (4), (5) and (6) the values of the coefficients a, b and c were inserted, $Q=1$ was assumed and the value of B was inferred. If the assumptions involved in the analysis are correct, the values of B derived in this way should agree generally with those adopted for the Moon. The values are plotted in the two figures, the first of which shows the comparison for the full period covered ; the second shows on a larger scale the comparison from 1835 onwards, the right ascension observations of the Sun and the observations of Venus

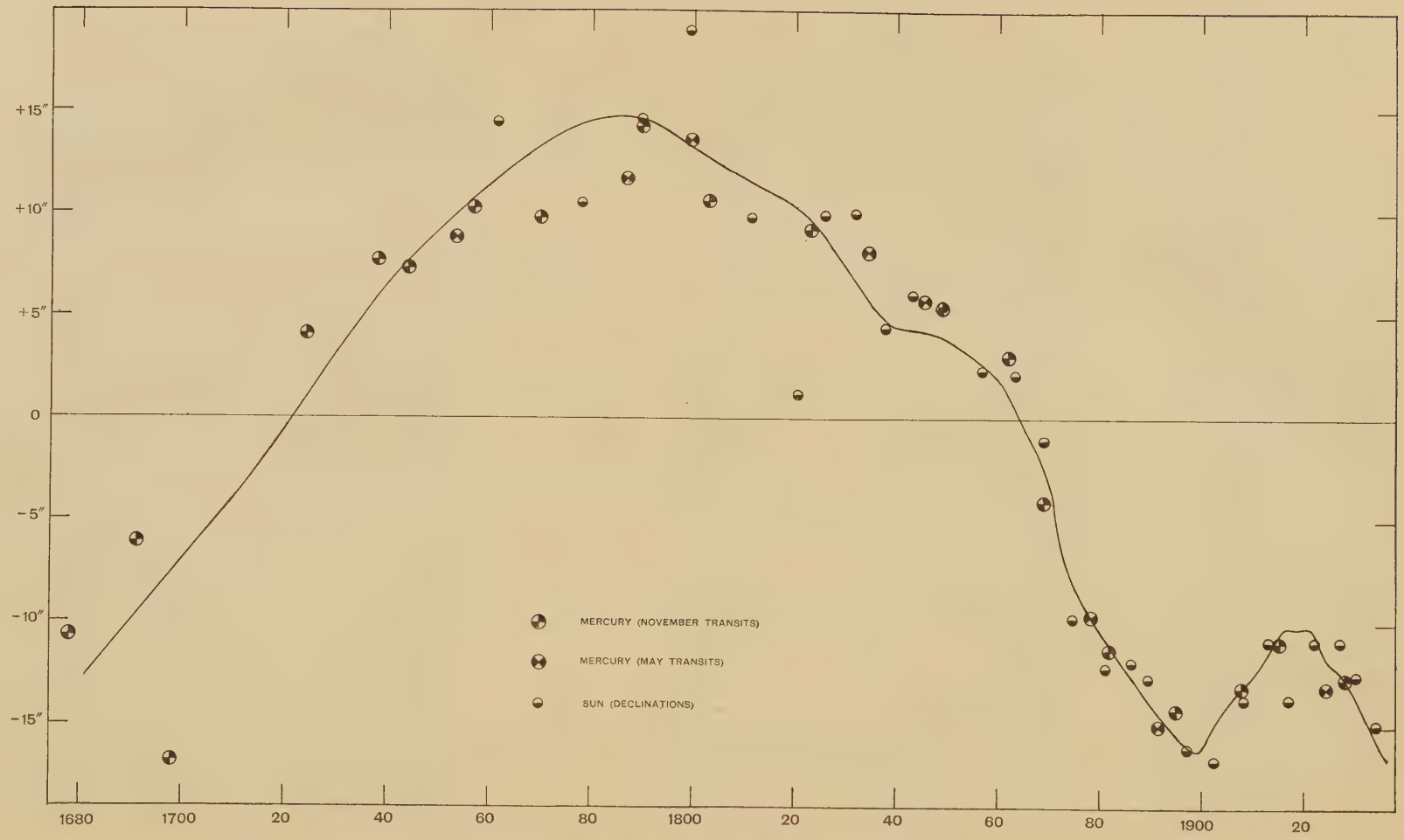


FIG. 1.—*Fluctuations in the longitude of the Moon from* 1680, *from observations of the Moon, Sun and Mercury.*

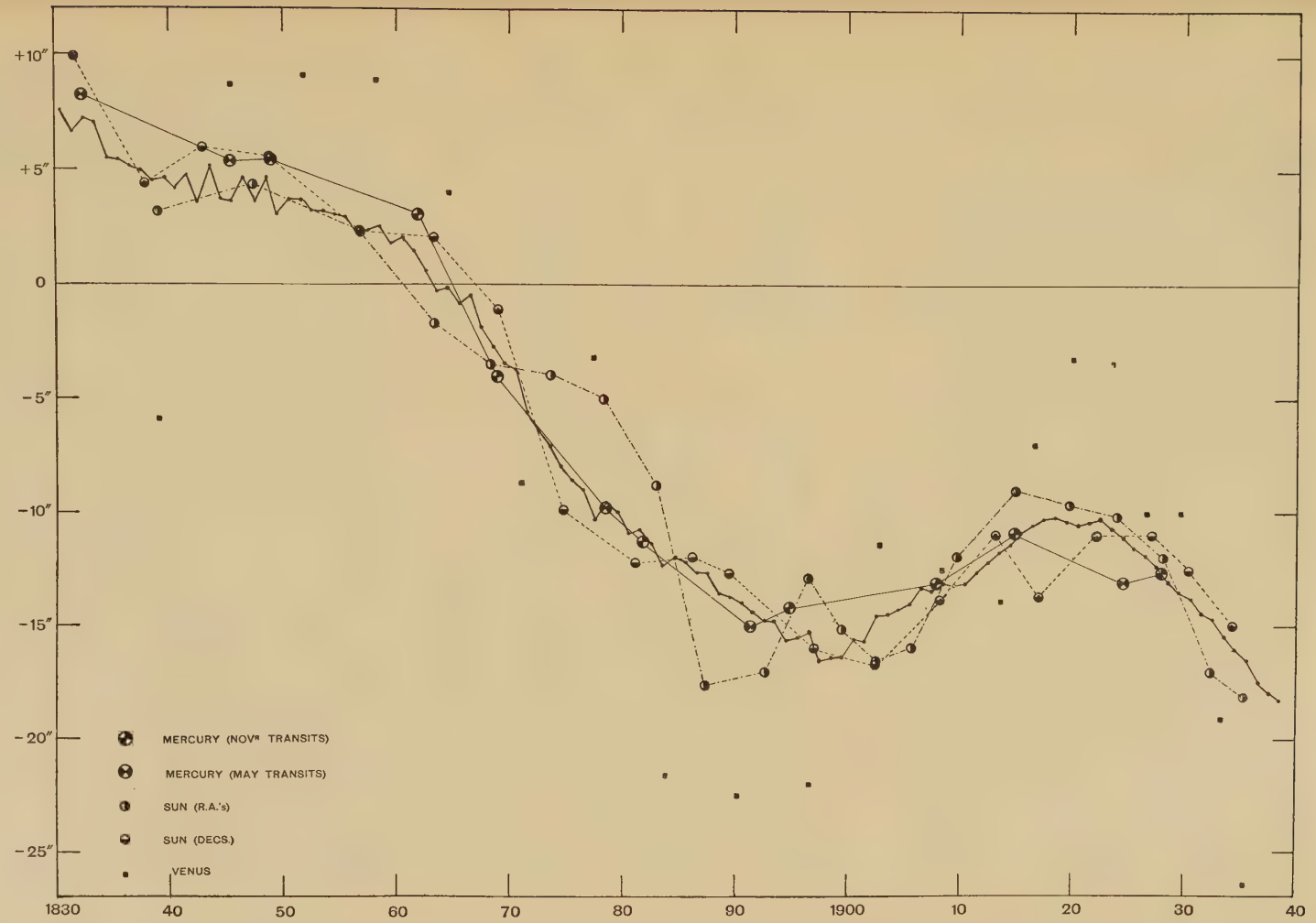


FIG. 2.—*Fluctuations in the longitude of the Moon, inferred from observations since* 1830 *of the Moon, Sun, Mercury and Venus.*

being restricted to this shorter period. The agreement is on the whole extremely good ; the chief discordances are shown in the Venus observations, but this is to be anticipated because B is determined from equation (6) with very low weight. In fig. 1 a smoothed representation of the Moon's fluctuations is drawn ; in fig. 2 observed values are given.

8. *The Secular Accelerations of the Sun and Moon.*—We have adopted a value of $5''\cdot22$ for the portion of the secular acceleration of the Moon that is due to tidal friction and have derived a value of $+1''\cdot23 \pm 0''\cdot04$ for the secular acceleration of the Sun.

This secular acceleration has been derived from the solution that combines all the material. It is of interest to examine to what extent the values given by the different series of observations are in agreement. Assuming, as before, that $Q = 1$ we obtain the following values :—

Sun	Declinations	$+1''\cdot19 \pm 0''\cdot11$
	Right Ascensions	$+0''\cdot47 \pm 0''\cdot41$
	Combined	$+1''\cdot14 \pm 0''\cdot11$
Mercury	November Transits	$+1''\cdot27 \pm 0''\cdot04$
	May	$+0''\cdot83 \pm 0''\cdot14$
	Combined	$+1''\cdot24 \pm 0''\cdot04$

The Venus observations have negligible weight for the determination of the secular acceleration.

The meridian observations of the Sun and the observations of the transits of Mercury are thus in close agreement.

From the ancient observations of solar and lunar eclipses and of occultations, as reduced and discussed by Fotheringham and collected and rediscussed by de Sitter *, the following values of the secular accelerations are derived as the most probable :—

For the Moon	$+5''\cdot22 \pm 0''\cdot30$,
For the Sun	$+1''\cdot80 \pm 0''\cdot16$.

This value for the secular acceleration of the Moon was adopted for the derivation of the fluctuations in the motion of the Moon used in the present discussion. The difference between the secular acceleration of the Sun derived from the ancient observations and that derived in the present discussion is about three times the sum of the probable errors of the two determinations. Though it is impossible to be certain, it seems probable that the difference is significant particularly in view of the close agreement between the values given by the Sun and Mercury. There is no *a priori* reason why the value derived in the present investigation should necessarily be in agreement with the value derived from the ancient eclipses ; the value derived from ancient eclipses represents a mean value during the past twenty centuries, whilst the value derived from the present discussion represents the mean value during the past two centuries or so. It is possible that the effects of tidal friction and the consequent retardation of the rotation

* *B.A.N.*, 4, 21, 1927, No. 124 (see § 2).

of the Earth have not remained absolutely constant. A slow progressive change in the effects of tidal friction seems to be not improbable.

We have assumed, however, for the secular acceleration of the Moon the value derived from the ancient eclipses. If tidal friction effects do not remain constant this assumption is not correct. Some assumption must necessarily be made, because otherwise the irregular fluctuations in longitude cannot be separated from the effects of the secular acceleration. We have implicitly assumed that during the period for which fairly reliable values of the longitude of the Moon are available, which we may take to be the last two hundred and fifty years, the longitude of the Moon has fluctuated by approximately equal amounts on either side of its mean value.

If we suppose that during the period covered by the present investigation the true secular acceleration of the Moon is not $+5''\cdot22$, as we have supposed, but $+5''\cdot22+s$, then we must replace B by a quantity B', which we may define by

$$B' = B - sS,$$

where $S = T^2 + 1\cdot3T - 0\cdot3$. The quantity S, which has zero values at 1750 and 1920, is introduced instead of T^2 in order to secure agreement with modern observations; in other words, it automatically takes account of the necessary adjustments to longitude at epoch and to mean motion.

The term in T^2 and B in the Sun and planets then becomes

$$\text{For the Sun,} \quad \frac{n_0}{n}(B - sS) + \left(c + \frac{n_0}{n}s\right)T^2,$$

$$\text{For the planets,} \quad \frac{n_i}{n_0}\left[\frac{n_0}{n}(B - sS) + \left(c + \frac{n_0}{n}s\right)T^2\right].$$

The analysis therefore proceeds as before, but corresponding to a secular acceleration for the Moon of $+5''\cdot22+s$ we will derive a secular acceleration for the Sun of amount $(c + n_0 s/n)$, which is equal to $+1''\cdot23 + (\cdot0747)s$.

Now, though there is nothing inherently improbable in the supposition that the effects of tidal friction are slowly variable, it seems valid to assume that the effects on the secular accelerations of the Moon and Sun remain in a constant ratio. The value of this ratio cannot be determined theoretically, because there are changes in some of the elements of the Moon's orbit that cannot be calculated and are too small to be checked by observation. It may be assumed, therefore, that at any time the ratio of the secular accelerations of the Moon and Sun caused by tidal friction is equal to the ratio of the average values over the last two thousand years, which can be inferred from the ancient eclipse and occultation observations.

With this assumption we have the relationship

$$\frac{1\cdot80 \pm \cdot16}{5\cdot22 \pm \cdot30} = \frac{(1\cdot23 \pm \cdot04) + \cdot0747s}{5\cdot22 + s},$$

which gives $s = -2\cdot11 \pm \cdot57$.

The best values that we can assign for the secular accelerations of the

Sun and Moon at the present time (or more strictly, the best average values for the past two hundred and fifty years) are therefore

$$\begin{aligned} &\text{For the Moon} \quad . \quad . \quad +3''\cdot11 \pm 0''\cdot57, \\ &\text{For the Sun} \quad . \quad . \quad . \quad +1''\cdot07 \pm 0''\cdot06. \end{aligned}$$

These values of the accelerations will not satisfy any of the ancient observations of eclipses and occultations, which are on the whole in very good agreement with one another in requiring appreciably larger values. There seems to be no escape from the conclusion that the effects of tidal friction are appreciably less at the present time than the average effects over the past two thousand years. There is no indication of the large and sudden fluctuations in tidal friction suggested by de Sitter *, and it is not necessary to assume that any sudden variations have taken place.

For the purpose of representing current observations by a formula, it is sufficient to retain B as defined in section 2 and the secular accelerations determined on this basis. The best possible separation of "fluctuations" from true secular acceleration is obtained, however, by adopting the smaller secular accelerations given above and assigning the remaining portion to the fluctuations.

9. *Corrections to Mean Longitudes of Sun, Mercury and Venus. The Sun.*—The declination observations give directly the correction to the true longitude of the Sun, $\Delta\lambda$. The right ascension observations give the corrections to the right ascension, $\Delta\alpha$, measured from the zero point of the clock-star system. We have obtained the following values :—

$$\begin{aligned} \Delta\lambda &= +1''\cdot00 + 2''\cdot97T + 1''\cdot23T^2 + \cdot0747B \\ &\quad \pm \cdot02 \pm \cdot05, \\ \Delta\alpha &= +1''\cdot28 + 2''\cdot69T + 1''\cdot23T^2 + \cdot0747B \\ &\quad \pm \cdot03 \pm \cdot12. \end{aligned}$$

The difference $\Delta\lambda - \Delta\alpha = E$, the equinox correction, or correction to the zero of the clock-star system, is

$$E = -0''\cdot28 \pm 0''\cdot04 + (0''\cdot28 \pm 0''\cdot13)T.$$

The right ascension observations have been reduced to the system of the Greenwich transit instrument with observations by the hand-tapping method. The systematic difference between the hand tapper and the impersonal micrometer that has been applied is $0''\cdot45$. Referring back to the impersonal micrometer, which is now generally used, we obtain

$$\begin{aligned} E &= -0''\cdot73 + 0''\cdot28T \\ &= -0^s\cdot048 + 0^s\cdot019T \\ &\quad \pm \cdot002 \pm \cdot009. \end{aligned}$$

T is measured in centuries from 1900. The correction for 1900 is in close agreement with other determinations. The correction to the motion of the equinox is known to be small; some determinations have given a

* *B.A.N.*, 4, 21, 1927 (see p. 35).

positive correction, others a negative. The correction to the motion here derived is about twice as great as its probable error. The balance of the evidence provided by other determinations is in favour of a small positive correction to Newcomb's motion.

Venus.—We have obtained

$$\Delta l_2 - \Delta L = +0''\cdot98 + 2''\cdot70T + 0''\cdot77T^2 + \cdot0471B \\ \pm \cdot04 \pm \cdot16.$$

The discussion has been based on right ascensions of Venus. For ΔL we may therefore use the value of $\Delta\alpha$ obtained above, giving

$$\Delta l_2 = +2''\cdot26 + 5''\cdot39T + 2''\cdot00T^2 + \cdot112B.$$

Mercury.—The following representations of V and W have been obtained :—

$$V = +5''\cdot81 + 16''\cdot01T + 6''\cdot35T^2 + \cdot385B, \\ W = +2''\cdot74 + 6''\cdot82T + 2''\cdot42T^2 + \cdot147B.$$

We may express V and W in the forms

$$V = 1\cdot487\Delta l_1 - 1\cdot01\Delta L + \delta V_0 + T\delta V_1, \\ W = 0\cdot716\Delta l_1 - 0\cdot97\Delta L + \Delta W_0 + T\delta W_1,$$

where the last two terms take account of corrections to other elements of the orbits of Mercury and the Earth.

Using the expressions for ΔL, Δl_1, given in (1) and (2) we obtain

$$1\cdot487(a' + b'T) + \delta V_0 + T\delta V_1 = +6\cdot82 + 19\cdot01T. \qquad (7)$$

$$\cdot716(a' + b'T) + \delta W_0 + T\delta W_1 = +3\cdot71 + 9\cdot70T. \qquad (8)$$

It is not possible to derive a', b', δV_0, δV_1, δW_0, δW_1 separately from these equations. In terms of the corrections to the elements e, π and e', π' of the orbits of Mercury and the Sun respectively, we have

$$\delta V_0 = -0\cdot487\delta\pi - 1\cdot137\delta e + 1\cdot19e'\delta\pi' + 1\cdot58\delta e', \\ \delta W_0 = +0\cdot284\delta\pi + 0\cdot896\delta e - 1\cdot11e'\delta\pi' - 1\cdot62\delta e'.$$

δV_1, δW_1 have similar expressions with $D_t\delta\pi$ for $\delta\pi$, etc.

The principal portions of the right-hand sides of (7) and (8) arise from the terms involving a' and b', the part contributed by the corrections to the orbital elements being relatively small. It will be seen that the signs of corresponding terms in δV_0 and δW_0 are opposite and that the coefficients are such that $3\delta V_0 + 4\delta W_0$ and $3\delta V_1 + 4\delta W_1$ are small. We therefore assume that we may neglect these quantities, in which the small contributions of the orbital element corrections are much reduced. We then obtain

$$7\cdot325(a' + b'T) = +35\cdot30 + 95\cdot83T,$$

whence

$$a' = +4''\cdot82, \qquad b' = +13''\cdot08.$$

The correction required by Newcomb's Tables of Mercury is therefore

$$\Delta l_1 = +4''\cdot96 + 13''\cdot08T + 5''\cdot10T^2 + \cdot310B.$$

We thus obtain as the corrections required to the mean longitudes of the Sun, Mercury and Venus, respectively, as given by Newcomb's Tables :

$$\begin{aligned}
\Delta L &= +1''\cdot00 + 2''\cdot97T + 1''\cdot23T^2 + \cdot0747B,\\
\Delta l_1 &= +4''\cdot96 + 13''\cdot08T + 5''\cdot10T^2 + \cdot310B,\\
\Delta l_2 &= +2''\cdot26 + 5''\cdot39T + 2''\cdot00T^2 + \cdot112B,
\end{aligned}$$

B denoting the fluctuation in the mean longitude of the Moon, as defined in section 2. T is expressed in centuries measured from 1900·0.

Summary.—The fluctuations in the mean longitudes of the Sun, Moon, Mercury and Venus are proportional to the respective mean motions of these bodies and can be attributed to variations in the rotation of the Earth, caused by changes in its moment of inertia. The secular accelerations of the Sun, Mercury and Venus are proportional to their mean motions and can be accounted for by retardation of the Earth's rotation by tidal friction. The secular acceleration of the Moon cannot be predicted theoretically, because tidal friction causes changes in the orbit of the Moon that are too small to be determined by observation.

The observations of the Sun and of Mercury agree in indicating that the average effects of tidal friction during the past two hundred and fifty years are smaller than the average effects during the past two thousand years. There is no indication of any large or sudden change.

Corrections to the mean longitudes of the Sun, Mercury and Venus, involving the fluctuation of the Moon's longitude, are derived. These expressions can be used for the accurate prediction of eclipses and transits of Mercury and Venus, when the tabular error of the Moon's longitude is known.

MOTION PICTURES OF SMALL CHROMOSPHERIC FLOCCULI

Robert R. McMath

The question of the life-period, size and general physical behaviour of solar granulations has received considerable attention in recent years. Omitting certain older researches, to which references are available in the papers mentioned, some recent investigations are:

H. H. Plaskett, "Solar Granulations", *M.N.*, **96**, 402, 1936.

P. ten Bruggencate und W. Grotrian, "Die Bestimmung der mittleren Lebensdauer der Granulation", *Z. f. Astroph.*, **12**, 323, 1936.

P. ten Bruggencate, W. Grotrian und E. v. d. Pahlen, "Ueber die Helligkeitsschwankungen und die Struktur der Sonnengranulation im Ultravioletten", *Z. f. Astroph.*, **16**, 1, 1938.

P. C. Keenan, "Dimensions of the Solar Granules", *Astroph. J.*, **88**, 360, 1938.

P. ten Bruggencate, "Beitrag zur Technik von Granulationsaufnahmen", *Z. f. Astroph.*, **16**, 374, 1938.

Several investigators have suggested from time to time that the fine structure of the chromosphere as depicted by the spectroheliograph is an extension upward of the granules illustrated and discussed in the papers to which reference has been made. It has been considered and quite generally accepted that a spectroheliogram made with a very narrow slit and precisely centred on the line in use depicts a relatively high level in the chromosphere; that a picture made using the wing of the line depicts a lower level than that of the centre of the line. From time to time efforts have been made to assign levels to various parts of the line upon theoretical considerations. The difference in appearance between spectroheliograms taken in the exact centre of the line and those with the line somewhat displaced on the second slit has long been recognized. Normally, the investigator has been concerned with the different parts of a prominence seen in projection on the disc rather than the fine structure of the chromosphere itself.

Reference to Plate 10 will show clearly that photospheric granules are not the subject of discussion. The upper half of Plate 10, A, is a high-level spectroheliogram, while B has been taken with the slit displaced one-half angstrom to the violet. The small dark markings well distributed throughout the area covered by the spectroheliogram are the features which show motion when the picture is projected.

On 1938 August 13, while taking motion pictures of a dark hydrogen flocculus at the McMath-Hulbert Observatory, H. E. Sawyer displaced the second slit one-half angstrom toward the violet to register a portion of the flocculus which showed velocity in the line of sight when examined in the spectrohelioscope. Sufficient exposures were made to provide an adequate "run" on the screen when the resulting spectroheliograms were shown by means of a motion-picture projector. Upon projection, the motions of the

small, dark flocculi are very clearly seen, and it was realized that a motion picture of the fine chromospheric detail had been secured.

On a later date the same experiment was repeated with the second slit displaced one-half angstrom toward the red; the resulting pictures show the same phenomena, but not so definitely. It has not yet been determined whether the difference between the red and the violet sides of the centre of the line is real or is due mostly to seeing effects; it may be mentioned at this point that very good seeing seems essential to obtain these films with success.

On the screen the "crawling" of the small dark flocculi is very clearly shown; the surface of the Sun resembles nothing quite so much as it does an ocean viewed directly from above, and covered completely with short and choppy billows in a constant progression of irregular growth and recession. The exposures for each spectroheliogram were 8 seconds, and five to six pictures per minute were made; the "compression factor" when projected on the screen is thus of the order of 1 : 80 or 1 : 120.

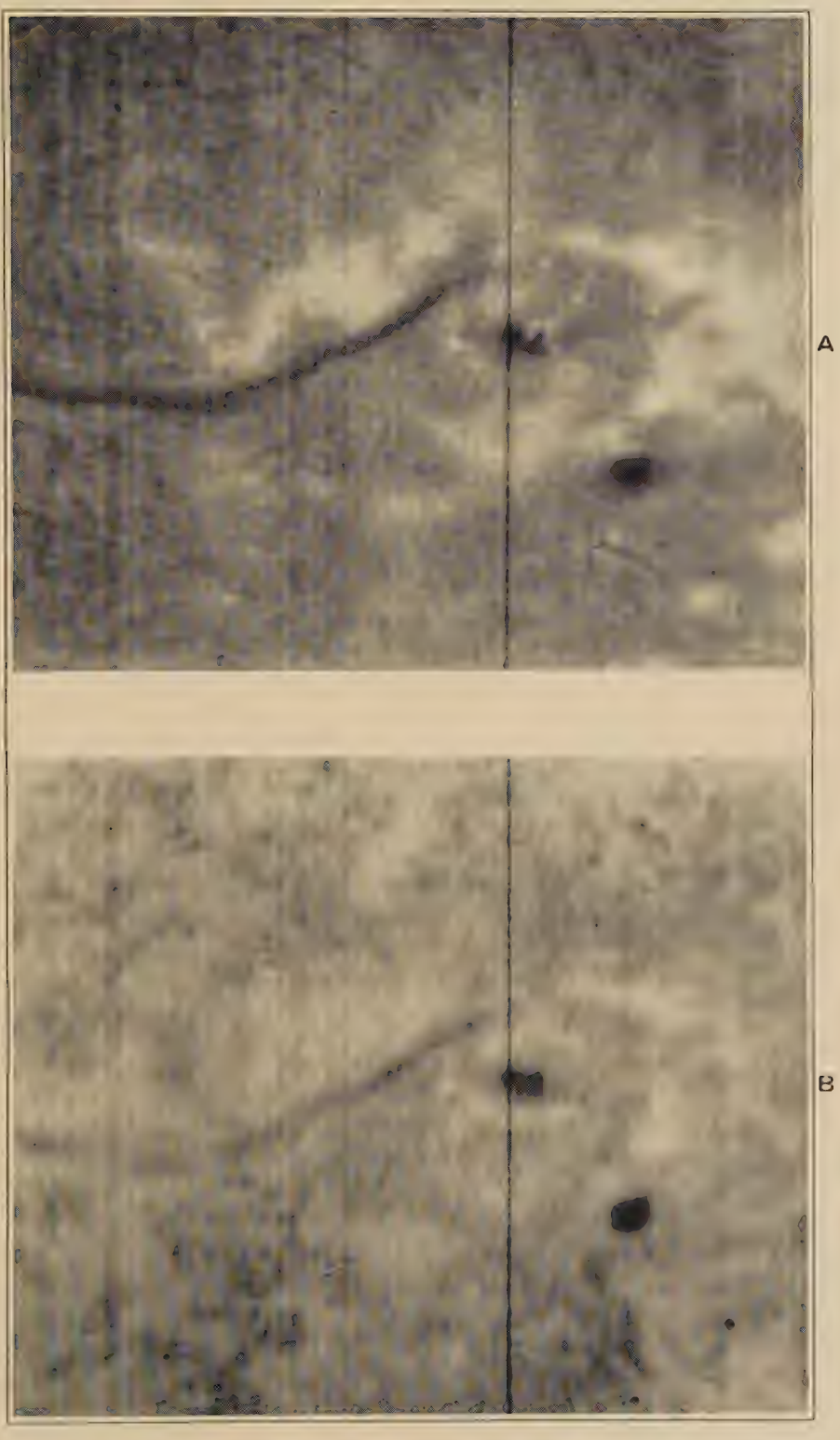
The very few astronomers who have thus far had an opportunity to see the motions of the small dark flocculi thus projected on the screen over a considerable area and "in bulk" have been delighted and astonished; one of them has stated that he had never seen anything that had brought home to him so vividly the truth that the surface of a star is in continuous turbulence and motion.

The approximate size of the markings is from 4″ to as large as 16″·5. It should be emphasized, of course, that the markings recorded are chromospheric and not photospheric granulations, inasmuch as they have been secured with a spectroheliograph.*

These spectroheliograms are believed to be the first taken at such a rate as to show clearly the motions involved. For an accurate determination, however, of the mean life-period of these flocculi in the statistical sense, we feel that accurate identification of the markings measured on successive motion-picture frames is the primary desideratum, as the total life is probably of the order of only a few minutes of time. Such accuracy of identification, as well as reliable data for any motions involved, requires a very considerably higher speed in taking the individual frames, perhaps as high as 8 per second for the most trustworthy results. We have accordingly made no effort to get out a preliminary value of the life-period from the material now available, but are planning to undertake this portion of the research this coming summer, after sufficient material is on hand taken at such higher speeds.

McMath-Hulbert Observatory of the
University of Michigan:
1939 *March* 20.

* As mentioned above and elsewhere in this note, we have deemed it necessary to lay emphasis on the fact that these motion pictures record chromospheric rather than photospheric phenomena. There is the very strong probability, however, felt by all who have seen these films, that these small chromospheric clouds will prove eventually to be essentially identical as to life-period, dimensions and motions with the well-known photospheric granulations, though considerable future research will be necessary before this identity can be definitely announced.



Motion Pictures of Small Chromospheric Flocculi. Plate A, above, is a high-level spectroheliogram, while the lower picture, B, was taken of the same region with the slit displaced one-half angstrom to the violet.

1 mm. = 2·9 ± 0·1 secs. arc.

R. R. McMath, Motion Pictures of Small Chromospheric Flocculi.

ON THE RELATION BETWEEN THE STREAM AND ELLIPSOIDAL CONSTANTS

W. M. Smart

1. Despite the apparent dissimilarity between the analytical forms of the two-streams and the ellipsoidal theories, it is almost invariably found in practice, when the numerical populations of the two streams are about equal, that the distribution of proper motions in any region of the sky can be equally well represented by either theory. Further, if we calculate the two theoretical distributions for a given region, using the appropriate constants of the two theories in turn, the two results are almost identical. It follows that the several constants involved must be connected in almost an exact way. Some years ago the writer obtained a formula * of this kind. The purpose of the present paper is to derive the values of the ellipsoidal constants using only the values of the stream-constants, and to compare the values so derived with those obtained from an analysis † of radial velocities which the writer recently made.

The formula in question is

$$\frac{1}{\epsilon^2} - 1 = h^2V_1^2 + h^2V_2^2 - 2h^2W^2 \sin^2 \lambda, \tag{1}$$

in which ϵ is the ratio of the minor to the major axis of the velocity ellipse for a given region, hV_1 and hV_2 are the projections of the drift-velocities on the tangent plane for the region (each in terms of the theoretical unit, $1/h$), hW is the solar motion and λ the angular distance of the region from the solar apex.

Further, if hU_1 and hU_2 denote the space-velocities of the drifts relative to the Sun, and λ_1 and λ_2 the angular distances of the region from the drift-apices, we have

$$hV_1 = hU_1 \sin \lambda_1, \qquad hV_2 = hU_2 \sin \lambda_2, \tag{2}$$

We write

$$q = \frac{1}{\epsilon^2} - 1, \tag{3}$$

and, for purposes of computation,

$$\tau_1 = hU_1 \cos \lambda_1, \qquad \tau_2 = hU_2 \cos \lambda_2, \tag{4}$$

so that (1) becomes

$$q = h^2U_1^2 + h^2U_2^2 - 2h^2W^2 - \tau_1^2 - \tau_2^2 + 2h^2W^2 \cos^2 \lambda. \tag{5}$$

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* *M.N.*, **89**, 114, 1928. See also the author's *Stellar Dynamics*, p. 248, 1938.

† *M.N.*, **99**, 61, 1938.

The right-hand side of this last formula involves the stream-constants; with these known we can calculate the value of q for any region of the sky.

Denoting the ratio of the minor to the major axis of the velocity ellipsoid by K/H and the angular distance of a region from the vertex of preferential motions by χ, we have the well-known ellipsoidal formula

$$q = \left(\frac{H^2}{K^2} - 1\right) \sin^2 \chi, \tag{6}$$

where q is defined by (3). The right-hand side of this equation involves the ellipsoidal constants; if q has been calculated for each region concerned by means of (5), the formula (6) is an equation of condition from which we are to determine the ellipsoidal constants.

2. In the paper last referred to, on the analysis of radial velocities on the ellipsoidal theory, the writer used 16 pairs of antipodal regions covering the whole sky except for the areas within 30° of the galactic poles. Although the total number of radial velocities used in this investigation is just about half the number of stars in Boss's *P.G.C.*, the results obtained from the radial velocities may be expected to be representative of the naked-eye stars as a whole. Eddington's analysis * of the proper motions of the Boss stars still remains the authoritative expression of the systematic motions of the naked-eye stars on the two-streams theory and, accordingly, his values of the stream-constants have been used in evaluating the right-hand side of (5) for each of the sixteen regions. It should be remarked, however, that, according to Eddington's analysis, the stars are not quite equally divided between the two streams, and so in a numerical application the formula (1) is rather more approximate than it would be on an equal distribution of the Boss stars between the two streams.

3. We write

$$\frac{H^2}{K^2} - 1 = x, \tag{7}$$

so that (6) becomes

$$x \sin^2 \chi = q. \tag{8}$$

Assuming that the vertex lies in the galactic equator in longitude G_0, χ is given by

$$\cos \chi = \cos (G - G_0) \cos g, \tag{9}$$

where (G, g) are the galactic co-ordinates of the centre of a region. Using (8) as an equation of condition, subject to (9), we have to determine the values of x and G_0. We proceed as follows:—

Let x_0 and G_0' be approximate values of x and G_0, and write

$$x = x_0 + \Delta x, \qquad G_0 = G_0' + \Delta G, \qquad \chi = \chi_0 + \Delta \chi,$$

the value of χ_0 being given by

$$\cos \chi_0 = \cos (G - G_0') \cos g. \tag{10}$$

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* *M.N.*, **71**, 4, 1910.

Then from (8) we have approximately

$$x_0 \sin^2 \chi_0 + \Delta x \sin^2 \chi_0 + 2x_0 \sin \chi_0 \cos \chi_0 \,.\, \Delta\chi = q. \qquad (11)$$

But, from (9), $\Delta\chi$ and ΔG are connected by

$$\Delta\chi \sin \chi_0 = \Delta G \sin (G - G_0') \cos g. \qquad (12)$$

From (10), (11) and (12) we have

$$\Delta x \sin^2 \chi_0 + \Delta G \, x_0 \sin 2(G - G_0') \cos^2 g = q - q_0, \qquad (13)$$

where

$$q_0 = x_0 \sin^2 \chi_0. \qquad (14)$$

We write (13) in the form

$$a\Delta x + b\Delta G = q - q_0, \qquad (15)$$

which is now our equation of condition ; here

$$a = \sin^2 \chi_0 ; \qquad b = x_0 \sin 2(G - G_0') \cos^2 g. \qquad (16)$$

4. According to Eddington's analysis,

$$hU_1 = 1{\cdot}516, \qquad hU_2 = 0{\cdot}855, \qquad hW = 0{\cdot}908.$$

Inserting these values in (5) we obtain

$$q = 1{\cdot}3804 - \tau_1^2 - \tau_2^2 + 1{\cdot}649 \cos^2 \lambda.$$

In a previous paper * the writer had occasion to calculate the values of τ_1 and τ_2, using Eddington's drift-velocities and the positions of the drift-apices ; these values are to be found in the paper referred to, and they are reproduced in Table I, which also contains the galactic co-ordinates

TABLE I

Region	Galactic Co-ordinates		τ_1	τ_2	cos λ
	G	g			
	°	°			
H	112½	0	0·341	−0·750	0·106
I	157½	0	1·250	−0·600	−0·556
$\mathcal{J}$	202½	0	1·425	−0·090	−0·894
K	247½	0	0·765	+0·466	−0·707
E	120	+20	0·358	−0·846	+0·139
F	180	+20	1·226	−0·486	−0·589
G	240	+20	0·731	+0·221	−0·578
L	120	−20	0·628	−0·571	−0·159
M	180	−20	1·494	−0·213	−0·887
N	240	−20	1·000	+0·496	−0·876
B	120	+45	0·119	−0·826	+0·300
C	180	+45	0·791	−0·546	−0·247
D	240	+45	0·405	+0·020	−0·240
P	120	−45	0·652	−0·278	−0·315
Q	180	−45	1·326	0·000	−0·863
R	240	−45	0·941	+0·555	−0·855

* *M.N.*, 96, 169, 1936.

of the centres of the sixteen regions. The last column of this table gives the corresponding values of cos λ derived from Eddington's position of the solar apex (in galactic co-ordinates this position is: 29°·3, +25°·8). The values of q calculated from (5) are shown in Table II.

TABLE II

Region	a	b	q	q_0	$q-q_0$	Residual (unit 0·001)
H	0·534	−1·367	0·720	0·732	−0·012	− 1
I	0·001	−0·096	0·000	0·002	−0·002	+ 1
$\mathcal{J}$	0·465	+1·367	0·659	0·637	+0·022	− 10
K	0·998	+0·096	1·402	1·367	+0·035	− 35
E	0·474	−1·188	0·568	0·649	−0·081	+ 69
F	0·225	+0·794	0·213	0·308	−0·095	+102
G	0·976	+0·394	1·349	1·337	+0·012	− 9
L	0·474	−1·188	0·702	0·649	+0·053	− 65
M	0·225	+0·794	0·399	0·308	+0·091	− 84
N	0·976	+0·394	1·400	1·337	+0·063	− 60
B	0·702	−0·672	0·832	0·962	−0·130	+123
C	0·561	+0·450	0·558	0·769	−0·211	+214
D	0·986	+0·223	1·311	1·351	−0·040	+ 41
P	0·702	−0·672	1·042	0·962	+0·080	− 87
Q	0·561	+0·450	0·850	0·769	+0·081	− 78
R	0·986	+0·223	1·393	1·351	+0·042	− 41

To calculate the coefficients a and b in the equation of condition (15) and also the values of q_0, we require suitable approximations to the values of x and G_0. The analysis * of the radial velocities had shown that for stars of types A to M (we limit ourselves to these spectral classes, as in Eddington's investigation the stars of Orion type were omitted) the values of K/H and G_0 were 0·697 and 339°·5 respectively; taking this value of K/H, the corresponding value of x is, by (7), 1·058. A preliminary solution taking 339°·5 as the definitive value of G_0, with $\Delta G = 0$, showed that this value of x was too low. After a further trial, the values of x_0 and G_0' were chosen as follows:—

$$x_0 = 1{\cdot}37, \qquad G_0' = 339^\circ{\cdot}5.$$

The coefficients a and b were then calculated together with the quantities q_0. These are shown in Table II together with the values of $q-q_0$. The normal equations formed from the equations of condition are as follows:—

$$7{\cdot}513\Delta x + 0{\cdot}003\Delta G = -0{\cdot}0084,$$
$$0{\cdot}003\Delta x + 9{\cdot}557\Delta G = +0{\cdot}0851,$$

from which

$$\Delta x = -0{\cdot}0011 \pm 0{\cdot}0217,$$
$$\Delta G = +0{\cdot}0089 \pm 0{\cdot}0192.$$

The residuals in the several equations of condition are given in the last column of Table II.

* *M.N.*, **99**, 68, 1938.

The solution-values of x and G_0 are then

$$x = 1{\cdot}3689 \pm 0{\cdot}0217,$$
$$G_0 = 339^\circ{\cdot}0 \pm 1^\circ{\cdot}1.$$

From (7) we find that

$$K/H = 0{\cdot}650 \pm 0{\cdot}003. \qquad (17)$$

These values of K/H and G_0 are thus the values of the ellipsoidal constants derived by means of (1) from a knowledge of the stream-constants alone.

5. It is rather remarkable that two such widely diverse methods, one the application of a formula involving the stream-constants and the other a direct analysis of radial velocities on the ellipsoidal hypothesis, should give practically identical results for the longitude of the vertex. It is to be remarked that this value is still about 12° higher than that found by Shapley from the distribution of globular clusters and by similar methods, and also higher by about the same amount than that deduced by various authors from the rotation-theory of the galaxy.

The values of K/H derived by the two methods are reasonably accordant when one considers the uncertainty of the value found in the analysis of the radial velocities. The almost negligible probable error in (17) is simply the evidence of the high degree of accuracy with which the formula (1) binds together the stream-constants with the ellipsoidal constants.

Table III gives the values of ϵ—the ratio of the minor to the major axis of the velocity ellipse—for each of the sixteen regions, obtained in three ways. The entries in columns 2 and 6 show the values obtained from the radial velocity results, namely, $K/H = 0{\cdot}697$, $G_0 = 339^\circ{\cdot}5$; columns 3 and 7 give the values calculated directly from formula (1) with Eddington's stream-constants; columns 4 and 8 give the values calculated from the results of the solution just obtained, namely, $K/H = 0{\cdot}650$, $G_0 = 339^\circ{\cdot}0$.

TABLE III

Values of ϵ

Region	Radial Velocities	Stream-constants: Formula (1)	Stream-constants: Final Solution	Region	Radial Velocities	Stream-constants: Formula (1)	Stream-constants: Final Solution
H	0·80	0·76	0·76	*M*	0·90	0·85	0·87
I	1·00	1·00	1·00	*N*	0·70	0·65	0·65
J	0·82	0·78	0·78	*B*	0·76	0·74	0·71
K	0·70	0·65	0·65	*C*	0·79	0·80	0·75
E	0·82	0·80	0·78	*D*	0·70	0·66	0·65
F	0·90	0·91	0·87	*P*	0·76	0·70	0·71
G	0·70	0·65	0·65	*Q*	0·79	0·74	0·75
L	0·82	0·77	0·78	*R*	0·70	0·65	0·65

The somewhat higher values in the second and seventh columns as compared with the remaining columns are, of course, the consequence of the higher value of K/H found from the radial velocities; the accordance

shown by columns 3 and 7 with columns 4 and 8 is a measure of the accuracy of formula (1).

Summary.—The paper deals with the connection between the stream-constants and the ellipsoidal constants. With Eddington's values of the stream-constants obtained from his analysis of the proper motions of the Boss stars, the ratio of the minor axis to the major axis of the velocity ellipsoid and the longitude of the vertex are found to be 0·650 and 339°·0. A comparison between the results deduced from these values and the results of an analysis based on radial velocities is given.

University Observatory,
University Gardens,
Glasgow, W. 2 :
1939 *April* 27.

ON THE FISSION OF A ROTATING FLUID MASS

Raymond A. Lyttleton, M.A., Ph.D.

A recent paper * by Lt.-Col. Edgeworth on the above subject would seem to necessitate some comment. The article can be divided into two parts ; in the first the author attacks my description of the outcome of the fissional process and in the second proposes an alternative solution. In neither part of the article does the argument seem to be guided by dynamical principles, and in addition to this there is a lack of accuracy of statement. For instance, it is claimed at the beginning of the article that it has been commonly accepted that fission divides the mass into two unequal parts. But the chief protagonist in the fission theory has described fission as resulting in nearly equal components † ; and it is only recently that attention has been drawn to the dynamical inconsistency of this view. Indeed, the requirement that the mass-ratio is greater than 3 : 1 has been one of the main points from which I have been led to advance the notion of complete separation of the masses resulting from fission.

As is stated in the article, in undergoing this ejection the material forming the smaller mass must have its motion changed from circular (just before instability sets in) to parabolic. In section (ii) of his paper Edgeworth selects a region B where this satellite is supposed to have parabolic velocity, and compares the velocity at this place with the velocity at A where the motion was circular. It is stated that the kinetic energy of the satellite when at B must be considerably greater than at A. This, however, is not necessarily true. The energy for escape varies inversely as the distance apart of the components, while the kinetic energy in circular motion is half the kinetic energy in parabolic motion at the same distance. From this it follows that with the figures given in the paper the amounts of energy possessed by the satellite at A and B are in the ratio $\frac{1}{2}(0{\cdot}8)^{-1} : (1{\cdot}3)^{-1} = 0{\cdot}8 : 1$, neglecting a slight correction arising from the mass of the satellite that reduces the second quantity in this ratio. The difference is certainly not alarming when it is remembered that the calculation may be in error due to (*a*) the fact that the numbers 0·8 and 1·3 are values chosen merely for illustration, and (*b*) the effects of the figures of the components on their mutual potential energy. This second feature must rise to importance.

In endeavouring to explain the supposed deficiency of energy Edgeworth centres interest on the point B, though there is no indication made how this point is selected. Nevertheless, supposing B were the place where the satellite first achieved its identity as a separate mass with parabolic velocity,

* *M.N.*, 99, 272, 1939.

† Jeans, *Problems of Cosmogony and Stellar Dynamics*, p. 252. Also see, Aitken, *George Darwin Lecture*, 1932, p. 608.

the force required, as drawn in his figure, is in the wrong direction by about 90°. The integrated force acting on the satellite must be parallel to the vector difference of the velocities at B and A, and for the figure given this difference is approximately perpendicular to the direction drawn by Edgeworth. When it is realised that the required force is in the radial direction its nature becomes evident. The only possible forces are those due to gravitation and pressure, and the total effect of the latter over the interface between the satellite and the main masses is seen to be in the radial direction. It is remarkable that no mention of the pressure is made in the article, considering the fact that the discussion supposedly relates to the forces that give rise to the motion of the fluid. It is reasonable to conjecture that the gravitational attraction on the material of the satellite diminishes as the figure elongates and this would facilitate the escape of the satellite.

Consider next the second section of the paper in which a new description of the process of fission is advocated. The mechanism advanced adopts as starting-point a suggestion made in 1928 by Jeans concerning the initial development of the pear-shaped figure. But at that time it was not generally recognised that the pear-shaped figure is ordinarily unstable; the belief that the pear-shaped figures would come into existence depended on the series being ordinarily stable and secularly unstable. In point of fact the pear-shaped series of figures does not come into existence at all. However, apart from this initial criticism, the details of the process of fission suggested in the article are open to serious question. It is not made clear why the spiral arm just extends to Roche's limit or why the velocity should be such that the tip of the arm does not come within the limit again and be disrupted. Furthermore, it is stated that the effect of the ellipsoidal form of the central mass on a particle moving within the Roche limit is to increase the angular momentum of the particle.

This description of fission contends that the final result consists of the formation of a satellite. The cosmogonical value of the speculation is questionable when it is recalled that no satellites can have been produced in this way, since even in the extreme case of the Earth-Moon system the total angular momentum is insufficient to render the combined mass unstable.

These few remarks relate only to the main features of the article and may perhaps be sufficient answer to the criticism therein advanced.

St. John's College, Cambridge :
1939 *April* 25.

TWELVE NEW SOUTHERN VARIABLES

J. B. G. Turner

A recent discovery of two new variable stars on a series of plates taken for parallax determination *, and the ease with which the discoveries were made, led me to believe that a further investigation of similar material might, notwithstanding the large scale of the plates, prove sufficiently productive of results to justify the labour involved. So far, twelve more new variables have been found and examined, the details of which are given below.

The plates used were all exposed through the Victoria telescope, aperture 24 inches and focal length 22 feet 6 inches; they are 16 cm. square and cover an area slightly more than one degree square. One millimetre = 30″·6. Three exposures of a uniform length of time are normally made at each observation. A series of parallax observations consists of upwards of twenty-five plates, divided more or less equally among the five epochs of observation. An epoch represents a length of time of at least four weeks; wherever possible, the observations are spread out over the entire period.

The method of detecting variables was as follows. From the series under investigation a plate of good definition was selected as standard, and the remaining plates of the series were in turn superposed upon it so as to bring the star images of the upper plate into proximity to—but not exact coincidence with—the corresponding images of the standard plate. The resulting pairs of images were then examined for evidence of variation. When a star was suspected of variability, comparison stars were selected as close to it as possible, their magnitudes established and the magnitudes of the variable determined throughout the series by comparing the size and density of its image on each plate with those of the comparison stars on the same plate.

The determination of the magnitudes of the comparison stars was at first a matter of some difficulty. The method originally employed was to identify a large number of stars, near the variable, in the *Astrographic Catalogue* (or, in one case, the C.P.D.) covering the region under review, and, using the magnitudes given for these stars as a starting-point, to smooth out by intercomparison the discrepancies found. The smoothed magnitudes thus obtained were then adopted in comparing the variable. It seemed probable, however, that while magnitudes so derived agreed sufficiently well among themselves, they represented values that were consistently too high, and an alternative method was therefore tried. Two plates were exposed through the glass, with exposures of $\frac{1}{2}$ min., 1 min., $1\frac{1}{2}$ min., 2 min., $2\frac{1}{2}$ min. and 3 min. on the Harvard Standard Region E4 in such

* *The Observatory*, **61**, 1938 December.

a way as to include all the stars fainter than magnitude 10 for which magnitudes have been determined. These stars were all identified and their magnitudes permanently inscribed on the two plates, of which one carried the exposures ½ min. to 2 min. and the other those of 2 min. to 3 min. In order to assign magnitudes to selected comparison stars in any particular series it was then necessary to identify on the first plate of that series—which for experimental purposes commonly receives six exposures ranging from ½ min. to 4 min.—as many stars as possible, fainter than magnitude 10, from the Henry Draper Catalogue, place it film to film against an E4 plate and establish a connection between the Harvard stars on the parallax plate and stars of the same photographic magnitudes on the E4 plate. Having identified particular exposures on the two plates with one another, by working down to the fainter stars, and transferring the identity, when necessary, by further inspection to the longer exposures, the magnitudes of the comparison stars were obtained with, it was felt, considerably greater accuracy. Of the stars given below, Nos. 4, 5, 10 and 12 were examined against comparison stars whose magnitudes were found in this way.

The following tables give the results obtained. Table I gives the

TABLE I

Summary of Results

No.	Constellation	R.A.	Dec.	Range	Remarks
		h m s	° ′		
1	Cetus	0 48 20	− 2 53	9·8–10·4	B.D. −2° 127. Short period
2	Eridanus	4 33 53	−14 29	9·7–10·4	Tacubaya −14° 4^h 32^m, No. 131
3	Orion	6 8 38	− 1 1	9·5–<14	Long period
4	Puppis	7 13 53	−41 8	12·2–14·0	Algol type?
5	Carina	9 39 48	−72 19	10·8–11·7	
6	Octans	12 19 59	−86 45	10·0–10·5	C.P.D. −86° 258
7	Ara	17 30 23	−52 10	10·4–11·4	
8	Ara	17 31 54	−51 49	11·1–11·7	
9	Ara	17 33 28	−52 18	10·0–10·4	C.P.D. −52° 10795
10	Telescopium	18 35 50	−46 20	12·5–13·6	Co.D. −46° 12584
11	Indus	21 9 29	−67 47	10·4–11·6	C.P.D. −67° 3787. Long period?
12	Grus	21 52 12	−51 6	12·3–14·5	Short period

number of the variable in the present series, its constellation, R.A. and Declination—these being for 1855 for stars north of −23° and for 1875 for stars south of that declination—the range of variation found and further remarks relative to the star, such as the *Durchmusterung* or *Astrographic Catalogue* number. Table II gives the results in detail. Under the serial number of the variable are listed the Julian Day Numbers for the plates on which it was examined and against these the estimated magnitudes. Below this there follows a summary of the dates on which the plates were secured. None of the variables here discussed appear in the Berlin-Babelsberg *Handbuch* for 1939.

TABLE II

Detailed Results

No. 1		No. 2		No. 3	
J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
2428457·43	10·3	2427416·42	9·8	2427436·44	< 14 *
490·34	10·2	2428448·61	10·2	2428486·57	< 11·5 †
492·33	9·9	451·58	10·3	496·55	< 11·5 *
503·30	9·8	456·57	10·2	497·54	< 13 *
506·30	10·1	457·57	10·2	563·35	10·6
507·29	10·2	458·56	10·3	571·32	10·8
508·28	10·0	461·56	10·2	577·31	< 11 †
719·71	10·2	463·56	10·1	588·29	11·1
722·70	9·9	533·37	10·2	591·27	< 11 †
723·69	10·2	538·36	10·2	595·27	< 11 †
729·67	10·0	542·35	10·2	832·61	< 11·5 †
730·68	10·3	546·34	10·1	852·55	< 11 †
731·66	10·3	561·30	10·1	854·54	11·1
733·66	9·9	562·29	10·1	855·55	11·0
849·35	10·1	564·29	10·1	863·53	10·8
850·35	9·9	791·66	10·2	864·52	10·7
855·33	10·0	799·64	10·3	924·36	< 11 †
857·32	10·1	807·60	10·4	925·37	< 11·5 †
861·30	10·0	808·60	10·3	930·34	< 11 †
2429081·70	10·4	812·59	10·3	939·32	< 11·5 †
092·68	10·2	817·58	10·3	951·29	< 11·5 †
094·68	10·1	821·58	10·3	959·26	< 11·5 †
100·65	9·9	897·38	9·7	2429196·62	9·5
102·65	10·0	904·35	9·8	198·62	9·5
215·34	10·0	906·34	9·7	209·58	10·0
218·34	10·1	910·33	9·7	223·54	10·5
226·32	10·0	926·29	9·7	232·52	10·8
227·30	10·0	928·29	9·8	287·36	< 11 †
228·31	10·0	2429158·65	10·4	289·36	< 11·5 †
240·29	9·9	160·65	10·4	299·34	< 11·5 †
		170·61	10·4	322·27	< 11·5 †
		182·58	10·4		
		185·57	10·2	* Just visible.	
		196·55	10·2	† Not seen, too faint.	

Epochs : 1936 Oct.–Dec.
1937 July
Nov.
1938 July
Nov.–Dec.

Epochs : 1933 Dec.
1936 Oct.
1937 Jan.
Sept.–Oct.
1938 Jan.
Sept.–Oct.

Epochs : 1933 Dec.
1936 Nov.
1937 Jan.–Mar.
Oct.–Nov.
1938 Jan.–Mar.
Oct.–Nov.
1939 Jan.–Feb.

TABLE II—*continued*

No. 4		No. 5		No. 6	
J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
2428496·59	12·2	2427497·41	11·1	2427584·31	10·3
497·57	13·7	516·36	10·9	595·29	10·2
498·56	12·3	559·25	11·0	610·24	10·5
502·56	12·2	560·24	11·1	612·23	10·3
503·56	12·2	563·21	11·2	614·24	10·0
507·54	12·3	566·22	11·4	617·21	10·4
583·34	12·4	567·21	11·2	621·22	10·5
585·33	12·3	800·58	10·8	622·21	10·2
587·32	12·3	809·56	11·0	833·63	10·1
602·28	12·4	816·55	11·3	834·63	10·0
608·27	12·4	817·55	11·1	837·62	10·0
612·26	12·4	916·27	11·2	854·56	10·0
854·59	12·3	927·23	11·3	861·55	10·1
855·58	12·2	930·23	11·4	964·27	10·2
863·56	12·4	931·22	11·5	973·25	10·0
868·56	12·4	934·21	11·5	981·23	10·2
875·54	12·3	938·20	11·4	985·19	10·2
877·54	12·3	2428154·61	11·5	2428198·63	10·3
939·36	12·2	159·60	11·5	205·62	10·1
940·35	12·3	174·56	11·7	208·61	10·3
951·33	12·3	176·56	11·5	212·59	10·0
952·33	12·3	177·54	11·5	215·58	10·2
958·31	12·3	182·55	11·5	218·58	10·5
976·26	12·3	281·28	11·0	328·28	10·3
984·24	12·3	286·27	11·0	331·27	10·2
2429218·59	14·0	287·27	10·9	336·27	10·2
221·59	12·3	296·24	11·0	344·24	10·0
223·59	12·2	298·24	11·0	349·22	10·3
227·57	12·2	311·20	11·3	567·63	10·3
237·54	12·3	315·20	11·3	574·61	10·2
322·30	12·3			577·59	10·4
324·31	12·2			581·59	10·2
325·30	12·3			584·58	10·1
344·25	12·3			586·57	10·2
348·24	12·2				
353·24	12·3				

Epochs : 1936 Nov.–Dec.
1937 Feb.–Mar.
Nov.–Dec.
1938 Feb.–Mar.
Nov.–Dec.
1939 Feb.–Mar.

Epochs : 1934 Feb.–May
Dec.
1935 Jan.
Apr.–May
Dec.
1936 Jan.
Apr.–May

Epochs : 1934 May–July
1935 Jan.–Feb.
June–July
1936 Jan.–Feb.
June
1937 Feb.

TABLE II—*continued*

	No. 7	No. 8	No. 9
J.D.	Mag.	Mag.	Mag.
2428409·25	11·3	11·2	10·1
410·24	11·2	11·2	10·1
414·23	11·2	11·3	10·3
417·23	11·2	11·7	10·1
623·67	11·3	11·3	10·1
624·66	11·3	11·2	10·1
636·62	11·3	11·2	10·2
650·59	11·2	11·2	10·2
758·29	11·2	11·4	10·1
762·28	11·3	11·2	10·1
764·27	11·4	11·2	10·2
774·23	11·3	11·4	10·1
779·23	11·3	11·4	10·2
985·67	11·1	11·4	10·2
992·64	*	11·2	10·1
2429002·62	10·9	11·2	10·2
010·60	11·0	11·6	10·1
124·30	10·4	11·2	10·0
126·29	10·5	11·1	10·2
131·28	10·7	11·5	10·3
134·26	10·9	11·2	10·1
137·25	10·9	11·2	10·1
150·23	11·1	11·6	10·4

* Fault on plate.

The above three variables were all discovered on the same series of plates.

Epochs : 1936 Aug.–Sept.
1937 Mar.–Apr.
Aug.–Sept.
1938 Mar.–Apr.
Aug.–Sept.

TABLE II—*continued*

No. 10		No. 11		No. 12	
J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
2428394·34	12·8	2428444·30	10·8	2428454·30	13·7
406·29	12·6	454·28	10·7	455·31	14·2
408·28	12·5	455·27	10·7	456·30	13·7
409·29	12·6	456·28	10·7	461·29	13·7
427·25	12·8	461·26	10·5	463·28	13·7
633·67	13·1	463·26	10·6	466·27	13·6
635·66	13·2	465·26	10·6	682·68	13·8
639·65	13·2	681·65	11·6	694·65	13·0
651·62	12·7	682·65	11·6	695·65	14·0
652·61	12·7	683·65	11·6	701·63	13·8
769·30	13·2	806·31	11·2	703·62	14·4
786·25	13·5	812·29	11·1	814·32	13·8
790·25	13·5	815·29	11·2	818·31	14·0
791·24	13·5	826·27	11·1	821·30	12·3
797·23	13·6	2429048·64	10·4	822·30	14·0
997·67	12·7	053·64	10·5	825·29	12·3

TABLE II—*continued*

No. 10		No. 11		No. 12	
J.D.	Mag.	J.D.	Mag.	J.D.	Mag.
2429017·62	13·3	2429055·63	10·5	2429046·69	12·7
022·60	13·2	056·63	10·7	048·68	12·5
029·57	13·2	070·58	10·8	055·65	14·0
134·30	12·8	170·33	10·9	056·66	12·9
137·30	12·7	173·31	11·0	063·62	14·5
141·28	12·7	180·29	11·0	064·63	13·0
149·27	13·1	185·28	11·0	070·61	13·0
157·24	12·6	187·27	11·0	173·34	13·6
				180·30	14·5
				182·31	14·5
				185·30	13·5
				194·28	13·9
				195·28	13·7

Epochs :	1936 Aug.–Sept.	Epochs :	1936 Oct.	Epochs :	1936 Oct.
	1937 Apr.		1937 May		1937 May–June
	Aug.–Sept.		Sept.–Oct.		Oct.
	1938 Apr.–May		1938 May–June		1938 May–June
	Aug.–Sept.		Sept.–Oct.		Oct.

I have to express my sincere gratitude to His Majesty's Astronomer at the Cape, Dr. J. Jackson, F.R.S., for permission to use the parallax material for the purpose of this investigation, and to Miss J. E. Still for valued assistance in the preparation of the results.

Royal Observatory,
Cape of Good Hope,
South Africa :
1939 *April* 25.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NIZAMIAH OBSERVATORY, HYDERABAD, DURING THE YEAR 1938

(Communicated by the Director)

No.	Star	Mag.	Phase	Date 1938	U.T. h m s
1	B.D. +15° 305	7·5	D	Jan. 10	18 51 0·5
2	54 Arietis	6·5	D	,, 11	20 2 3·5
3	ι Tauri	4·7	D	,, 13	13 32 29·2
4	105 Tauri	6·0	D	,, 13	16 20 54·1
5	ω Leonis	5·5	R	,, 18	0 16 46·2
6	29 Sagittarii	5·4	R	,, 29	0 42 46·3
7	36 Arietis	6·5	D	Feb. 7	16 7 13·0
8	B.D. +21° 707	6·9	D	,, 9	18 31 9·4
9	B.D. −20° 4364	5·9	R	,, 22	0 10 59·8
10	16 Geminorum	6·1	D	Mar. 10	13 59 18·3
11	A.G. Berlin 2412	8·3	D	,, 10	14 37 32·3
12	ν Geminorum	4·1	D	,, 10	14 41 52·6
13	29 Cancri	5·9	D	,, 12	15 21 13·4
14	ω Leonis	5·5	D	,, 13	18 53 3·8
15	ψ Virginis	4·9	R	,, 17	16 40 13·4
16	B.D. −19° 4135	5·5	R	,, 20	19 37 56·8
17	ω^2 Scorpii	4·6	D	Apr. 17	20 5 5·9
18	1 Cancri	6·0	D	May 5	14 44 34·7
19	A.G. Berlin 3116	8·7	D	,, 5	14 45 36·5
20	B.D. +16° 1591	9·5	D	,, 5	14 46 26·2
21	60 Cancri	5·7	D	,, 6	16 11 4·7
22	B.D. −6° 3557	7·1	D	,, 10	17 36 40·4
23	49 Virginis	5·3	D	,, 11	13 24 8·7
24	β Capricorni	3·2	R	Oct. 3	17 9 20·5
25	B.D. −14° 5732	6·7	D	,, 3	17 58 30·0
26	B.D. −14° 5734	7·0	D	,, 3	18 37 44·7
27	43 Tauri	5·7	R	,, 12	22 29 39·1
28	B.D. −13° 5773	6·0	D	,, 31	15 41 49·0
29	51 Aquarii	5·8	D	Nov. 2	15 30 11·8
30	B.D. +16° 349	9·2	D	,, 7	21 10 52·3
31	B.D. +16° 352	9·3	D	,, 7	21 24 19·6
32	Bordeaux +17° 2^h 44^m 122		D	,, 7	21 39 49·1
33	B.D. +16° 357	9·5	D	,, 7	21 54 20·2
34	Bordeaux +17° 2^h 44^m 130		D	,, 7	22 2 37·3
35	Bordeaux +17° 2^h 44^m 129		D	,, 7	22 12 0·7
36	B.D. +15° 393	9·5	D	,, 7	22 15 18·2
37	Bordeaux +17° 2^h 44^m 132		D	,, 7	22 34 8·2
38	Bordeaux +17° 2^h 44^m 136		D	,, 7	22 47 16·0
39	B.D. +16° 361	9·0	D	,, 7	23 1 55·6
40	30 Aquarii	5·6	D	,, 29	13 20 16·5
41	B.D. +10° 171	6·9	D	Dec. 3	18 37 2·2
42	B.D. +3° 4935	9·3	D	,, 29	13 30 33·0
43	B.D. +3° 1	9·3	D	,, 29	13 42 27·3
44	A.G. Albany 8	8·4	D	,, 29	14 49 24·0

Notes

No. 10 : time somewhat uncertain.
No. 14 : disappearance gradual.
No. 16 : seen through thin clouds.
No. 17 : Moon was frequently obscured by thick clouds ; time slightly uncertain.
Nos. 30–39 : observed during the total eclipse of the Moon on November 7.

Thirteen observations (Nos. 18 to 20 and Nos. 30 to 39) were made by T. P. Bhaskaran and the rest by M. K. Bappu. The instrument used was the 15-inch visual refractor, except in the case of No. 10 which was observed with the 3-inch finder. The clock corrections were determined from observations with the transit instrument.

The reductions were carried out by M. V. V. Sastri and S. Aravamudan. As in the previous year, a correction of $0^h.0015$ has been added to the observed time to allow for the error of 3″ in the mean longitude of the Moon.

No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$	No.	$\sigma' - \sigma$	$\cos(\rho - \chi)$	$\sin(\rho - \chi)$
	″				″		
1	+0·2	+1·00	+0·08	23	−2·2	+0·80	+0·60
2	+0·4	+ ·39	− ·92	24	+2·0	− ·84	− ·55
3	+0·2	+ ·61	− ·79	25	−2·9	+ ·78	+ ·62
4	−0·6	+ ·65	− ·76	26	−2·1	+ ·63	+ ·78
5	+1·5	− ·87	− ·49	27	−0·8	− ·98	− ·18
6	+1·3	− ·94	+ ·34	28	−1·3	+ ·97	− ·23
7	+0·6	+1·00	+ ·10	29	−0·3	+1·00	·00
8	−0·4	+ ·82	− ·57	30	+0·6	+ ·93	+ ·38
9	+1·5	− ·92	+ ·40	31	+1·0	+ ·96	+ ·26
10	−1·9	+ ·87	+ ·49	32	−1·8	+ ·78	+ ·62
11	+0·5	+ ·97	− ·26	33	−2·2	+ ·99	− ·13
12	−1·0	+ ·94	− ·34	34	[−6·8]	+ ·98	− ·19
13	−0·0	+ ·93	− ·37	35	+0·5	+ ·76	+ ·64
14	+1·0	+ ·21	− ·98	36	−3·5	+ ·70	− ·72
15	+0·9	− ·72	+ ·69	37	−1·3	+ ·54	+ ·84
16	+1·7	− ·60	+ ·80	38	+3·8	+ ·96	+ ·28
17	−2·4	+ ·99	+ ·17	39	+1·4	+ ·72	+ ·70
18	−1·1	+1·00	·00	40	−2·1	+ ·56	+ ·83
19	+0·8	+ ·99	+ ·15	41	−2·4	+ ·76	+ ·65
20	−0·1	+1·00	− ·08	42	+0·1	+1·00	+ ·07
21	−0·9	+1·00	+ ·02	43	−2·5	+ ·96	− ·27
22	−1·1	+ ·90	+ ·43	44	−2·0	+ ·53	+ ·85

The mean places of sixteen stars in the above list, for 1938·0, are given in the following table. For stars A.G. Berlin 3116 and A.G. Albany 8, proper motions have been applied to the co-ordinates in the catalogue, in computing the mean places for use in the reductions.

No.	R.A. (1938·0)	Dec. (1938·0)	Remarks
	h m s	° ′ ″	
11	6 25 12·96	+20 16 49·2	
19	7 53 31·84	+15 59 38·3	P.M. μ_α +0^s·0041 ; μ_δ −0″·124
20	7 53 29·97	+15 55 47·9	Bordeaux +16° 7^h 52^m, No. 80 ; +17° 7^h 48^m, No. 305

No.	R.A. (1938·0)	Dec. (1938·0)	Remarks
	h m s	° ′ ″	
30	2 44 36·20	+16 28 29·9	Bordeaux +16° 2^h 40^m, No. 58 ; +17° 2^h 44^m, No. 113
31	2 45 2·74	+16 27 52·2	Bordeaux +16° 2^h 40^m, No. 60 ; +17° 2^h 44^m, No. 116
32	2 45 11·89	+16 34 3·6	
33	2 46 1·79	+16 23 48·4	Bordeaux +16° 2^h 40^m, No. 67 ; +17° 2^h 44^m, No. 127
34	2 46 16·51	+16 23 27·6	
35	2 46 8·17	+16 36 32·7	Bordeaux +16° 2^h 40^m, No. 68 ; +16° 2^h 48^m, No. 5 ; +17° 2^h 44^m, No. 129
36	2 46 28·48	+16 14 57·3	Bordeaux +16° 2^h 40^m, No. 69 ; +16° 2^h 48^m, No. 6 ; +17° 2^h 44^m, No. 131
37	2 46 32·30	+16 40 16·5	Bordeaux +16° 2^h 48^m, No. 7 ; +17° 2^h 44^m, No. 132
38	2 47 33·73	+16 33 34·9	Bordeaux +16° 2^h 48^m, No. 10 ; +17° 2^h 44^m, No. 136
39	2 47 41·17	+16 40 13·5	Bordeaux +16° 2^h 48^m, No. 11 ; +17° 2^h 44^m, No. 138
42	0 4 8·22	+ 4 12 47·6	Alger +3° 0^h 4^m, No. 36 ; +4° 0^h 0^m, No. 102
43	0 4 25·98	+ 4 9 44·0	Alger +3° 0^h 4^m, No. 39 ; +4° 0^h 0^m, No. 227
44	0 4 55·11	+ 4 34 57·1	P.M. μ_α −0^s·0032 ; μ_δ −0″·051

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